Women's Liberation:
What's in It for Men?
Mathematical Appendix

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Abstract

This mathematical appendix contains the proofs of the lemmas and propositions in Doepke and Tertilt (2009).[1] Equations are numbered consecutively with the equations in the published paper; equation numbers up to (16) refer to the main text.

[1]Doepke, Matthias and Michèle Tertilt, "Women's Rights: What's in It for Men?" *Quarterly Journal of Economics*, forthcoming.

Proof of Proposition 1: We would like to derive a condition under which $V_m^E > V_m^P$, which in the no-education case is equivalent to:

$$u(c_m^E, c_f^E, n^E) + \frac{\gamma_m}{1 - \frac{\gamma_m + \gamma_f}{2}} \left[\frac{u(c_m^E, c_f^E, n^E) + u(c_f^E, c_m^E, n^E)}{2} \right]$$
$$> u(c_m^P, c_f^P, n^P) + \frac{\gamma_m}{1 - \frac{\gamma_m + \gamma_f}{2}} \left[\frac{u(c_m^P, c_f^P, n^P) + u(c_f^P, c_m^P, n^P)}{2} \right].$$

Plugging in the functional form for $u(\cdot)$ and the solutions for c_m^E, c_m^P, c_f^E, c_f^P, n^E, and n^P yields:

$$(1+\sigma) \log \left(\frac{1+\sigma}{2} \right) + \frac{\gamma_m}{1 - \frac{\gamma_m + \gamma_f}{2}} (1+\sigma) \log \left(\frac{1+\sigma}{2} \right)$$
$$> \sigma \log(\sigma) + \frac{\gamma_m}{1 - \frac{\gamma_m + \gamma_f}{2}} \frac{1+\sigma}{2} \log(\sigma)$$

or:

$$[2 - \gamma_f + \gamma_m](1+\sigma) \log \left(\frac{1+\sigma}{2} \right) > [(2 - \gamma_f)\sigma + \gamma_m] \log(\sigma). \tag{17}$$

Isolating the terms involving γ_m on the left-hand side gives:

$$\gamma_m \left((1+\sigma) \log \left(\frac{1+\sigma}{2} \right) - \log(\sigma) \right) > (2 - \gamma_f) \left(\sigma \log(\sigma) - (1+\sigma) \log \left(\frac{1+\sigma}{2} \right) \right).$$

For $\sigma = 1$, both sides are equal to zero, so that men are indifferent between the two regimes. For $0 < \sigma < 1$, both sides are strictly positive. Moreover, the left-hand side is strictly increasing in γ_m. Thus, if we define:

$$\bar{\gamma}_m = \frac{(2 - \gamma_f) \left(\sigma \log(\sigma) - (1+\sigma) \log \left(\frac{1+\sigma}{2} \right) \right)}{(1+\sigma) \log \left(\frac{1+\sigma}{2} \right) - \log(\sigma)},$$

we have that for all $\gamma_m > \bar{\gamma}_m$ inequality (17) is satisfied, implying that men prefer the empowerment regime E. Turning to the role of σ, note that both sides of (17) are strictly increasing in σ. However, as σ approaches zero the left-hand side converges to $-[2 - \gamma_f + \gamma_m] \log(2)$, whereas the right-hand side approaches minus infinity. Therefore, there exists a $\bar{\sigma}$ such that (17) is satisfied for all σ satisfying $0 < \sigma < \bar{\sigma}$. □

Proof of Lemma 1: We want to derive the equilibrium value functions for the case of positive education under the patriarchy and empowerment regimes. The proof is by guess and verify. We guess that the value functions take the form:

$$\begin{aligned}
V_m^P(H_m, H_f, \bar{H}) &= a_1^P + a_2 \log(H_m) + a_3 \log(H_f) + a_4 \log(\bar{H}_m) + a_5 \log(\bar{H}_f),\\
V_f^P(H_m, H_f, \bar{H}) &= b_1^P + b_2 \log(H_m) + b_3 \log(H_f) + b_4 \log(\bar{H}_m) + b_5 \log(\bar{H}_f),\\
V_m^E(H_m, H_f, \bar{H}) &= a_1^E + a_2 \log(H_m) + a_3 \log(H_f) + a_4 \log(\bar{H}_m) + a_5 \log(\bar{H}_f),\\
V_f^E(H_m, H_f, \bar{H}) &= b_1^E + b_2 \log(H_m) + b_3 \log(H_f) + b_4 \log(\bar{H}_m) + b_5 \log(\bar{H}_f).
\end{aligned}$$

By plugging these parameterized value functions into the right-hand sides of equations (6) and (7), we can derive explicit solutions for the individual choices, which are given in equations (15) and (16) in the text. Then, plugging the functional forms for the value functions, the optimal individual choices, and the laws of motion for human capital into both sides of the functional equation (8) yields a system of equations that can be solved for the value-function coefficients. The solutions for the slope coefficients (which are identical in the two political regimes) are:[2]

$$\begin{aligned}
a_2 &= \frac{(1+\sigma)[2(1-\alpha) - (1-\alpha)\beta\gamma_f + \alpha(1-\beta)\gamma_m]}{2-(1-\beta)\gamma_m - \beta\gamma_f},\\
a_3 &= (1+\sigma)\left(\alpha + \frac{\beta\gamma_m}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right),\\
a_4 &= \left(\frac{(1-\beta)\frac{\gamma_m}{2}}{1-\gamma_m/2-\gamma_f/2}\right)\left(\frac{(1+\sigma)[2+(1-2\beta)(\gamma_f-\gamma_m)]}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right),\\
a_5 &= \left(\frac{\beta\frac{\gamma_m}{2}}{1-\gamma_m/2-\gamma_f/2}\right)\left(\frac{(1+\sigma)[2+(1-2\beta)(\gamma_f-\gamma_m)]}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right),\\
b_2 &= (1+\sigma)\left((1-\alpha) + \frac{(1-\beta)\gamma_f}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right),\\
b_3 &= \frac{(1+\sigma)(2\alpha + (1-\alpha)\beta\gamma_f - \alpha(1-\beta)\gamma_m)}{2-(1-\beta)\gamma_m - \beta\gamma_f},\\
b_4 &= \left(\frac{(1-\beta)\frac{\gamma_f}{2}}{1-\gamma_m/2-\gamma_f/2}\right)\left(\frac{(1+\sigma)[2+(1-2\beta)(\gamma_f-\gamma_m)]}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right),\\
b_5 &= \left(\frac{\beta\frac{\gamma_f}{2}}{1-\gamma_m/2-\gamma_f/2}\right)\left(\frac{(1+\sigma)[2+(1-2\beta)(\gamma_f-\gamma_m)]}{2-(1-\beta)\gamma_m - \beta\gamma_f}\right).
\end{aligned}$$

[2] Step-by-step derivations are available on request.

The level coefficients in the two political regimes $j \in \{P, E\}$ can be expressed as:

$$
\begin{aligned}
a_1^j &= \frac{2-\gamma_f}{2-(\gamma_m+\gamma_f)}(M_1^j + M_2^j) + \frac{\gamma_m}{2-(\gamma_f+\gamma_m)}(F_1^j + F_2^j), \\
b_1^j &= \frac{\gamma_f}{2-(\gamma_f+\gamma_m)}(M_1^j + M_2^j) + \frac{2-\gamma_m}{2-(\gamma_m+\gamma_f)}(F_1^j + F_2^j),
\end{aligned}
$$

where:

$$
\begin{aligned}
M_1^P &= \sigma \log(\sigma) + (1+\sigma)\log\left(\frac{A}{1+\sigma}\left(\frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}\right)^{\alpha}\right) + \delta \log\left(\frac{\delta - \frac{\gamma_m}{2}(a_2+b_3)\theta}{\phi(\alpha(1+\sigma)+\delta)}\right), \\
M_2^P &= \frac{\gamma_m}{2}\theta \log(a_2)[a_2+a_4+b_2+b_4] + \frac{\gamma_m}{2}\theta \log(b_3)[a_3+a_5+b_3+b_5] \\
&\quad + \frac{\gamma_m}{2}\theta[a_2+a_3+a_4+a_5+b_2+b_3+b_4+b_5]\log\left(B\left(\frac{\phi\frac{\gamma_m}{2}\theta}{\delta-\frac{\gamma_m}{2}(a_2+b_3)\theta}\right)\right), \\
F_1^P &= \log(\sigma) + (1+\sigma)\log\left(\frac{A}{1+\sigma}\left(\frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}\right)^{\alpha}\right) + \delta \log\left(\frac{\delta - \frac{\gamma_m}{2}(a_2+b_3)\theta}{\phi(\alpha(1+\sigma)+\delta)}\right), \\
F_2^P &= \frac{\gamma_f}{2}\theta \log(a_2)[a_2+a_4+b_2+b_4] + \frac{\gamma_f}{2}\theta \log(b_3)[a_3+a_5+b_3+b_5] \\
&\quad + \frac{\gamma_f}{2}\theta[a_2+a_3+a_4+a_5+b_2+b_3+b_4+b_5]\log\left(B\left(\frac{\phi\frac{\gamma_m}{2}\theta}{\delta-\frac{\gamma_m}{2}(a_2+b_3)\theta}\right)\right), \\
M_1^E &= (1+\sigma)\log\left(\frac{A}{2}\left(\frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}\right)^{\alpha}\right) + \delta \log\left(\frac{\delta - \frac{\gamma_m+\gamma_f}{4}(a_2+b_3)\theta}{\phi(\alpha(1+\sigma)+\delta)}\right), \\
M_2^E &= \frac{\gamma_m}{2}\theta \log(a_2)[a_2+a_4+b_2+b_4] + \frac{\gamma_m}{2}\theta \log(b_3)[a_3+a_5+b_3+b_5] \\
&\quad + \frac{\gamma_m}{2}\theta[a_2+a_3+a_4+a_5+b_2+b_3+b_4+b_5]\log\left(\frac{B\phi\frac{\gamma_m+\gamma_f}{4}\theta}{\delta-\frac{\gamma_m+\gamma_f}{4}(a_2+b_3)\theta}\right), \\
F_1^E &= (1+\sigma)\log\left(\frac{A}{2}\left(\frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}\right)^{\alpha}\right) + \delta \log\left(\frac{\delta - \frac{\gamma_m+\gamma_f}{4}(a_2+b_3)\theta}{\phi(\alpha(1+\sigma)+\delta)}\right), \\
F_2^E &= \frac{\gamma_f}{2}\theta \log(a_2)[a_2+a_4+b_2+b_4] + \frac{\gamma_f}{2}\theta \log(b_3)[a_3+a_5+b_3+b_5] \\
&\quad + \frac{\gamma_f}{2}\theta[a_2+a_3+a_4+a_5+b_2+b_3+b_4+b_5]\log\left(\frac{B\phi\frac{\gamma_m+\gamma_f}{4}\theta}{\delta-\frac{\gamma_m+\gamma_f}{4}(a_2+b_3)\theta}\right).
\end{aligned}
$$

□

Proof of Proposition 2: All parts of the proposition follow from comparing the closed-form solutions for consumption, education, and fertility in both regimes (see (15) and

(16)) under the condition $\gamma_f > \gamma_m$. Aggregate consumption is:

$$C^P = C^E = A\Big(\frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}H_f\Big)^{\alpha}H_m^{1-\alpha}.$$

The fraction of time women spend on production is $t_f^P = t_f^E = \frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}$. Since the remaining time is spent on child care, total child care time is independent of the regime. That fertility is lower and education is higher under empowerment and that both of these choices are independent of state variables follows directly from the closed-form solutions given in (15) and (16). One implication of these findings is that the total time women devote to educating children is higher under empowerment, even though they have fewer children in this regime. Total female education time under patriarchy is $n^P(e_m^P+e_f^P) = \frac{\theta\gamma_m(1+\sigma)}{(\alpha(1+\sigma)+\delta)[2-(1-\beta)\gamma_m-\beta\gamma_f]}$, compared to $n^E(e_m^E+e_f^E) = \frac{\theta\gamma(1+\sigma)}{(\alpha(1+\sigma)+\delta)[2-(1-\beta)\gamma_m-\beta\gamma_f)}$ under empowerment. The gender education gap is given by $\frac{e_f}{e_m} = \frac{2\alpha+(1-\alpha)\beta\gamma_f-\alpha(1-\beta)\gamma_m}{2(1-\alpha)-(1-\alpha)\beta\gamma_f+\alpha(1-\beta)\gamma_m}$ in both regimes. Finally, the growth rate of aggregate consumption (and output and human capital) is given by $B^{\theta}(e_f^j)^{\theta\beta}(e_m^j)^{\theta(1-\beta)}$. Since, as argued above, $e_f^E > e_f^P$ and $e_m^E > e_m^P$, it follows that the growth rate is higher under empowerment. □

Proof of Proposition 3: Men will vote for empowerment if and only if their utility under empowerment exceeds the utility under patriarchy:

$$V_m^E(H_m, H_f, \bar{H}) > V_m^P(H_m, H_f, \bar{H}).$$

We have already determined that $V_m^E(H_m, H_f, \bar{H})$ and $V_m^P(H_m, H_f, \bar{H})$ differ only in the constant term, so that the inequality can be written as $a_1^E > a_1^P$. Writing out this condition and simplifying gives:

$$\begin{aligned}(2-\gamma_f+\gamma_m)(1+\sigma)\log\left(\frac{1+\sigma}{2}\right) &- [(2-\gamma_f)\sigma+\gamma_m]\log(\sigma)\\ &+\theta\gamma_m\frac{2(1+\sigma)}{(1-\gamma)}\log\left(\frac{\gamma}{\gamma_m}\right)+\left[\theta\gamma_m\frac{2(1+\sigma)}{(1-\gamma)}-(2-\gamma_f+\gamma_m)\delta\right]\\ &\times\log\Big(\frac{\delta[2-(1-\beta)\gamma_m-\beta\gamma_f]-\gamma_m(1+\sigma)\theta}{\delta[2-(1-\beta)\gamma_m-\beta\gamma_f]-\gamma(1+\sigma)\theta}\Big) > 0. \quad (18)\end{aligned}$$

The first line of this expression reflects the preference for equality in future generations that was already present in the no-education case (compare to inequality (17) in the proof of Proposition 1 above). The remaining terms reflect the role of education. As one would expect, setting $\theta = 0$ reduces the expression to the no-education case. Define $\theta^\star$

as:

$$\theta^{\star} = \frac{\delta[2-(1-\beta)\gamma_m - \beta\gamma_f]}{\gamma(1+\sigma)}. \tag{19}$$

Note that as θ approaches $\theta^{\star}$ from below, the denominator in the log term goes to zero and, hence, the log term goes to infinity. Further, the assumption $\gamma_m > \frac{\gamma_f}{3}$ assures that for θ sufficiently close to $\theta^{\star}$ the term in square brackets is strictly positive, so that the overall expression goes to plus infinity. Intuitively, if $\theta = \theta^{\star}$, parents can achieve any positive utility level by choosing a sufficiently small number of children with a sufficiently high level of education. Given that the left-hand side of (18) approaches plus infinity for θ sufficiently close to $\theta^{\star}$, there has to be a threshold $\bar{\theta}$ such that (18) is satisfied for all θ that satisfy $\bar{\theta} < \theta < \theta^{\star}$. Hence, for sufficiently high θ men will prefer empowerment over patriarchy. $\square$

Proof of Proposition 4: After plugging $\gamma_m = \gamma_f$ into (18), the condition for preferring equal rights reduces to

$$(2-\gamma_f+\gamma_m)(1+\sigma)\log\left(\frac{1+\sigma}{2}\right) > [(2-\gamma_f)\sigma+\gamma_m]\log(\sigma),$$

which is independent of θ and in fact identical to the condition for the no-education case. To show that the human capital externality is crucial for our results, we solve a version of the model without this externality, which is equivalent to assuming that sons and daughters marry each other. Since in this setup different dynasties do not interact, average human capital is no longer a state variable. The male and female value functions $i \in \{m, f\}$ in the two regimes $j \in \{P, E\}$ satisfy the following recursive relationship:

$$V_i^j(H_m, H_f) = u_i(c_m, c_f, n) + \frac{\gamma_i}{2}\left[V_m^j(H_m', H_f') + V_f^j(H_m', H_f')\right].$$

As before, choices are determined either by maximizing the male value function (patriarchy) or the average value function (empowerment). The value functions can be solved explicitly, and the condition under which men prefer empowerment is:

$$(2-\gamma_f+\gamma_m)(1+\sigma)\log\left(\frac{1+\sigma}{2}\right) - [(2-\gamma_f)\sigma+\gamma_m]\log(\sigma) + \gamma_m\frac{2(1+\sigma)}{1-\gamma}\theta\log\left(\frac{\gamma}{\gamma_m}\right)$$
$$+\left[\gamma_m\frac{2(1+\sigma)}{1-\gamma}\theta - \delta(2-\gamma_f+\gamma_m)\right]\log\left(\frac{\delta[1-\gamma]-\gamma_m(1+\sigma)\theta}{\delta[1-\gamma]-\gamma(1+\sigma)\theta}\right) > 0. \tag{20}$$

The maximum θ for which the problem is well defined is $\frac{\delta(1-\gamma)}{\gamma(1+\sigma)}$. Analogously to the proof of Proposition 3, the last logarithmic term goes to infinity in the limit. However, the expression multiplying the log term is negative for all θ less or equal to the limit. Since all other terms are finite, it follows that for large enough θ the expression is negative. Hence, men prefer the patriarchy regime for sufficiently large θ. □

Proof of Proposition 5: Under dynamic voting, a vote for empowerment in a given period T shifts the consumption allocation between husbands and wives at time T in favor of the wives, it lowers the fertility rate at time T, and it leads to an increase in all future human capital levels by the factor:

$$\left(\frac{e^E_{m,T}}{e^P_{m,T}}\right)^{\theta_T} = \left(\frac{e^E_{f,T}}{e^P_{f,T}}\right)^{\theta_T} = \frac{\gamma[\delta - \frac{\gamma_m}{2}(a_2 + b_3)\theta_T]}{\gamma_m[\delta - \frac{\gamma}{2}(a_2 + b_3)\theta_T]}.$$

Future decisions on the relative consumption allocation, fertility, and education are not affected by the vote. By plugging the decisions under votes for empowerment and patriarchy, respectively, into the male utility function and taking the difference (where most terms drop out), we find that men will vote for empowerment in period T if:

$$\begin{aligned} &2(1+\sigma)\log\left(\frac{1+\sigma}{2}\right) - 2\sigma\log(\sigma) \\ &\qquad + 2\theta_T\gamma_m\frac{(1+\sigma)}{1-\gamma}\log\left(\frac{\gamma}{\gamma_m}\right) + 2\left[\theta_T\gamma_m\frac{(1+\sigma)}{1-\gamma} - \delta\right] \\ &\qquad\qquad \times \log\left(\frac{\delta[2-(1-\beta)\gamma_m - \beta\gamma_f] - \gamma_m(1+\sigma)\theta_T}{\delta[2-(1-\beta)\gamma_m - \beta\gamma_f] - \gamma(1+\sigma)\theta_T}\right) > 0. \end{aligned} \tag{21}$$

This condition is similar to inequality (18) that was derived in the proof of Proposition 3, and the arguments of that proposition can also be applied here to show that there exists a threshold $\tilde{\theta}$ such that (21) is met for all θ_T that satisfy $\tilde{\theta} < \theta_T < \theta^\star$, where $\theta^\star$ is defined in equation (19). Hence, for sufficiently high θ_T men will vote for empowerment. Moreover, comparing condition (18) in Proposition 3 with condition (21) above, we find that in (21) the constant term (i.e., the first line) as well as the factor multiplying the logarithmic term in the last line are lower than in condition (18), which implies that the threshold $\tilde{\theta}$ is higher than the threshold $\bar{\theta}$ derived in Proposition 3, i.e., $\tilde{\theta} > \bar{\theta}$. □

Proof of Proposition 6: Recall that the male utility can be expressed as:

$$u(c_m, c_f, n) + \frac{\gamma_m}{2}\left[a_1 + a_2 \log(H'_m) + a_3 \log(\bar{H}'_f) + a_4 \log(\bar{H}'_m) + a_5 \log(\bar{H}'_f)\right]$$
$$+ \frac{\gamma_m}{2}\left[b_1 + b_2 \log(\bar{H}'_m) + b_3 \log(H'_f) + b_4 \log(\bar{H}'_m) + b_5 \log(\bar{H}'_f)\right]. \quad (22)$$

Dropping all constants (i.e., additive terms that do not depend on the policy variables τ and s), the political objective function is:

$$(1+\sigma)(1-\alpha)\log(1-\tau) + \frac{\gamma_m}{2}\sum_{i=2}^{5}(a_i + b_i)(1-\eta)\hat{\theta}\log(s).$$

Plugging in the budget constraint of the education system, this is:

$$(1+\sigma)(1-\alpha)\log(1-\tau) + \frac{\gamma_m}{2}\sum_{i=2}^{5}(a_i + b_i)(1-\eta)\hat{\theta}\log\left(\frac{S\tau}{2n}\right).$$

The first-order condition for choosing τ gives:

$$\frac{(1+\sigma)(1-\alpha)}{1-\tau} = \frac{\gamma_m}{2}\sum_{i=2}^{5}(a_i + b_i)\frac{(1-\eta)\hat{\theta}}{\tau}$$

The optimal tax rate therefore is:

$$\tau = \frac{\frac{\gamma_m}{2}\sum_{i=2}^{5}(a_i + b_i)(1-\eta)\hat{\theta}}{\frac{\gamma_m}{2}\sum_{i=2}^{5}(a_i + b_i)(1-\eta)\hat{\theta} + (1+\sigma)(1-\alpha)}.$$

Plugging in the solutions for the coefficients a_2 to a_5 and b_2 to b_5 gives the tax rate stated in the proposition. This tax rate applies in either political regime. Notice, however, that n depends on the regime, thus, through the effect on n, schooling per student s is higher under empowerment. □

Proof of Proposition 7: Men will vote for empowerment if and only if their utility under empowerment exceeds the utility under patriarchy:

$$V_m^E(H_m, H_f, \bar{H}) > V_m^P(H_m, H_f, \bar{H}).$$

We have already determined that $V_m^E(H_m, H_f, \bar{H})$ and $V_m^P(H_m, H_f, \bar{H})$ differ only in the constant term, so that the inequality can be written as $a_1^E > a_1^P$. Writing out this condition and simplifying gives:

$$
\begin{aligned}
&\frac{\gamma_m}{2-(\gamma_m+\gamma_f)}\hat{\theta}(1-\eta)[a_2+a_3+a_4+a_5+b_2+b_3+b_4+b_5] \\
&\qquad\left(\log\left(\delta-\frac{\gamma_m}{2}(a_2+b_3)\theta\right)-\log\left(\delta-\frac{\gamma_m+\gamma_f}{4}(a_2+b_3)\theta)\right)\right) \\
&\qquad+(2-\gamma_f+\gamma_m)(1+\sigma)\log\left(\frac{1+\sigma}{2}\right)-[(2-\gamma_f)\sigma+\gamma_m]\log(\sigma) \\
&\qquad+\theta\gamma_m\frac{2(1+\sigma)}{(1-\gamma)}\log\left(\frac{\gamma}{\gamma_m}\right)+\left[\theta\gamma_m\frac{2(1+\sigma)}{(1-\gamma)}-(2-\gamma_f+\gamma_m)\delta\right] \\
&\qquad\qquad\times\log\left(\frac{\delta[2-(1-\beta)\gamma_m-\beta\gamma_f]-\gamma_m(1+\sigma)\theta}{\delta[2-(1-\beta)\gamma_m-\beta\gamma_f]-\gamma(1+\sigma)\theta}\right)>0.
\end{aligned}
$$

This expression is identical to the condition given in Proposition 3 except for the first term (the first two lines). The new term is non-negative, and thus unambiguously increases the incentive to introduce empowerment relative to the case without public schooling. Moreover, the term is monotonically increasing in θ, and converges to zero as θ approaches zero from above. Thus, for sufficiently low θ patriarchy is still preferred, as long as it is preferred for low θ without public schooling. $\square$

Proof of Proposition 8: Female education is: $e_f^i = \frac{\phi\frac{\gamma_m}{2}b_{3i}\theta}{\delta-\frac{\gamma_m}{2}(a_{2i}+b_{3i})\theta}$ where $i = H, L$ refers to two economies with high and low α. Using the expressions from the proof of Lemma 1, $a_{2i}+b_{3i}$ reduces to $\frac{2(1+\sigma)}{2-(1-\beta)\gamma_m-\beta\gamma_f}$, which does not depend on α. Hence, the ratio reduces to $\frac{e_f^H}{e_f^L} = \frac{b_{3H}}{b_{3L}}$. Plugging in for b_3 and simplifying we have: $\frac{e_f^H}{e_f^L} = \frac{\alpha_H[2-\beta\gamma_f-(1-\beta)\gamma_m]+\beta\gamma_f}{\alpha_L[2-\beta\gamma_f-(1-\beta)\gamma_m]+\beta\gamma_f}$. The numerator is larger than the denominator and thus $e_f^H > e_f^L$. Using the same logic, it is easy to show that $e_m^H < e_m^L$. Average education time per child is equal to $\frac{1}{2}(e_f+e_m) = \frac{\phi\gamma_m\theta(b_3+a_2)}{\delta-\frac{\gamma_m}{2}(a_2+b_3)\theta}$ which does not depend on α. The time that women devote to market work is $t_f = 1-n(\phi+e_f+e_m) = \frac{\alpha(1+\sigma)}{\alpha(1+\sigma)+\delta}$, which increases in α. The wage per unit of time is w_fH_f for female and w_mH_m for males. Using the fact that wages equal the marginal product of (each type of) labor and plugging in the optimal time spent working, the wage ratio is: $\frac{w_fH_f}{w_mH_m} = \frac{\alpha(1+\sigma)+\delta}{(1-\alpha)(1+\sigma)}$, which is increasing in α. Optimal fertility is $n_i = \frac{\delta-\frac{\gamma_m}{2}(a_{2i}+b_{3i})\theta}{\phi(\alpha_i(1+\sigma)+\delta)}$ where as above $i = H, L$ indicates whether variables relate to an economy with a high or low α. Noting again that $a_{2i}+b_{3i}$ is independent of α, the fertility ratio simplifies to $\frac{n_H}{n_L} = \frac{\alpha_L(1+\sigma)+\delta}{\alpha_H(1+\sigma)+\delta}$. Therefore $n_H < n_L$. $\square$

Proof of Proposition 9: The result that the optimal regime is independent of α follows from condition (18), in which α does not appear. $\square$