Childless Aristocrats. Inheritance and the extensive margin of fertility*

Paula Gobbi† Marc Goñi‡

March 15, 2018

Abstract

We provide new evidence on the two-way link between inheritance and fertility. We focus on settlements, a scheme combining primogeniture and a one-generation entail of family estates. Using peerage records (1650-1882), we find that settlements reduced childlessness by 14.7 pp., ensuring the survival of aristocratic dynasties. Since settlements were signed only if the family head survived until his heir's wedding, we establish causality by exploiting variation in heir's birth order. Next, we present a model of inheritance with intergenerational hyperbolic discounting. We show that schemes that restrict successors (settlements, trusts...) emerge endogenously in response to concerns over the dynasty's survival.

Keywords: Childlessness, Inheritance, Elites, Settlement, Fertility, Inter-generational discounting.
JEL Classification Numbers: J13, K36, N33.

*We thank the suggestions made by David de la Croix, Daniel García, Moshe Hazan, Hannah Liepmann, Stefania Marcassa, Eeva Mauring, and Alessandro Sommacal and the seminar participants in Bordeaux, Durbuy, Essen, CEPR Macro-Growth (Frankfurt), Helsinki, Louvain-la-Neuve, Luxembourg, Odense, Université Libre Bruxelles, Université Saint-Louis-Bruxelles, Lund, SAET (Faro), Vienna, Wurzburg, SAEe (Barcelona). The Cambridge Group for the History of Population and Social Structure kindly shared data.
†Université libre de Bruxelles (ECARES) and CEPR
‡Department of Economics, University of Vienna

1 Introduction

Inheritance practices have long attracted the attention of economists. For example, Adam Smith gave a scathing criticism of primogeniture and entailment of the land: He argued that these laws exacerbated inequality, "making beggars" of all but the first-born.[1] This intuition has carried over to modern work. Several studies argue that inheritance practices have important effects for fiscal policy (Barro 1974), inequality (Stiglitz 1969; Chu 1991; Piketty 2011), or economic growth and the transition to modern, democratic societies (Bertocchi 2006).

However, what effect inheritance has on inequality, social mobility, or economic growth depends crucially on fertility choices. For example, a standard implication of models of intergenerational transfers is that if the very rich have more children, inheritances seemingly reduce inequality (see, e.g., Stiglitz 1969; Atkinson and Harrison 1978). Despite the central role of fertility, one common feature in the analysis of inheritance is to treat fertility as *exogenous* or consider endogenous fertility decisions only on the *intensive margin*—i.e., the number of children. In contrast, the interactions between inheritance rules and the *extensive margin* of fertility—i.e., the decision to have children or not—remain unexplored. This is surprising as the economic effects of any inheritance scheme and, in particular, of primogeniture or entailment, crucially hinge on the production of an heir.

In this paper we analyze inheritance and the extensive margin of fertility in a unified framework. Specifically, we show that inheritance schemes can have a strong, causal impact on the extensive margin of fertility and we develop a theory showing that, in turn, inheritance systems can emerge endogenously in response to concerns over the survival of a dynasty. Our analysis focuses on settlements, a popular inheritance scheme in Britain that combined primogeniture and a one-generation entail of the land. We first show that settlements crucially increased the extensive margin of fertility for British aristocrats. Using genealogical data between 1650 and 1882 we find that families signing a settlement were c. 15 percentage points more likely to have children. Given that the average childlessnes rate among peers was 17 percent, settlements increased by 83.5 percent the extensive margin of fertility, pushed childlessness rates close to the "natural" rate of 2.4 percent (Tietze 1957),[2] and hence, contributed

[1] Smith 1776 [1937], book III, chapter II.

[2] The "naural" rate corresponds to that of Hutterites, who marry young, do not divorce, have access to modern health care, etc.

to the survival of noble family lineages. In contrast, we find that settlements did not affect the intensive margin of fertility—the number of children by mothers. We focus on this historical setting for four main reasons. First, settlements had to be renewed by each generation at the time of the marriage of the heir (Bonfield 1979). Thus, when the father died (exogenously) before the marriage of his eldest son, it generated as good as a random assignment of families into settlements. To establish causality, we exploit exogenous variation in the probability that a father dies before his heir's wedding coming from the birth order of the heir. In our context, it is unlikely that birth order had a direct effect on later fertility, for example, through breastfeeding (Jayachandran and Kuziemko 2011). The reason is that women in the aristocracy typically hired wet nurses to breastfeed their children (Fildes 1986: 193). Second, a unique feature of our historical setting allows us to conduct placebo tests to validate our results. Unlike settlements in England and Ireland, Scottish entails were perpetual, i.e., they did not had to be renewed upon the heir's marriage (Habakkuk 1994: 6). We estimate our IV model for a comparable sample of women who should not be affected by settlements because they did not marry an heir, or because they married a Scottish heir. Our estimates are close to zero and significantly different from our benchmark results, suggesting that our model captures the effect of settlements and not other confounding factors. Third, while aristocrats ruled England between c.1550-c.1880 (Allen 2009) and today remain among the very rich in the UK, they were not pre-destined to the top of the distribution. Strong demographic pressures threatened the extinction of these lineages around the 1600s: around 40 percent of all married women in the aristocracy were childless. We show that settlements were crucial for the survival of the aristocracy in Britain as they moved them to a high fertility regime. Fourth, modern studies of inhertiance practices are typically restricted to primogeniture vs. equal sharing. However, settlements—or, more generally, land entails—were an important inheritance system, as highlighted by contemporaries like Adam Smith, Tocqueville, or Karl Marx. Today, inheritance schemes that restrict successors (e.g., trusts) are also widespread, especially among the top one percent (Wolff and Gittleman 2014).

In sum, the first result of the paper is that settlements moved the British aristocracy to a higher fertility regime. This implies that settlements contributed to the perpetuation of elite lineages not only by entailing the land or favoring primogeniture, but also through changing fertility incentives.

The second contribution of the paper is to show theoretically that concerns over the production of heirs and the survival of a dynasty can endogenously shape inheritance practices. To that end, we propose a general model where inheritance schemes that restrict successors' powers to manage inherited wealth (e.g., settlements or trusts) emerge as an outcome of the family head's concerns over the survival of the dynasty and the heir's optimal decision. Specifically, we model three generations of the same dynasty that decide sequentially over consumption, bequests, and fertility. We then compare the results of a benchmark model where every generation decides the bequest of the next generation (e.g., primogeniture) to a model with settlements—or, more generally, any inheritance scheme that restricts successors. We model the latter as a commitment device that allows the father to decide the bequests of the next three generations. We depart from standard models of inheritance in two ways: First, while inheritance practices are typically treated as exogenous (see Chu 1991 and references therein), we endogenize the use of inheritance schemes that restrict successors.[3] Second, we depart from the standard assumption of exponential discounting by assuming that individuals have a quasi-hyperbolic discount function towards future generations.

The model provides three sets of results: First, it rationalizes the existence of inheritance schemes that restrict successors—e.g., settlements or trusts. Second, the model replicates our empirical results and identifies the mechanisms behind it. Third, and most importantly, our model endogenizes the emergence of inheritance practices as an outcome of the family head's concerns over the survival of the dynasty and the heir's optimal decisions. That is, it also explains why an heir may agree to sign a settlement, and hence, renounce to freely dispose of the family wealth.

The first result is a byproduct of hyperbolic discounting across generations. This discounting implies that individuals do not value their children's well-being significantly more than that of the future generations, namely their grandsons. Fathers then have an incentive to limit their sons' discretion to manage the family estate, as this resolves this inter-generational time inconsistency. To our knowledge, we are the first to rationalize the existence of inheritance rules that restrict successors with hyperbolic discounting preferences. The classic overlapping generations model of bequests assumes exponential discounting across generations (Barro 1974). Our results show that such time consistent preferences across generations are hard to reconcile with

[3]Notable exceptions are (Chu 1991) and (Grieco and Ziebarth 2015). They show that primogeneiture can emerge endogenously as a result of economic factors.

inheritance practices in which the family head limits his successors' powers to manage the family wealth; like settlements (England), trusts, fee tails (United States), entails (Scotland), *majorat* (France), *mayorazgo* (Spain), or *ordynacja* (Poland).

Secondly, our model is able to replicate our empirical results and to identify imperfect altruism as the mechanism behind it. Specifically, we assume that altruism is higher towards direct descendants than towards distant relatives and show that the family dynasty is less likely to die out in the model with settlements than in the benchmark model where every generation decides the bequests of the next generation. The economic intuition is simple: when the second generation is subject to a settlement, he cannot appropriate the bequest set for generation 3 (e.g., by selling parts of the family estate). In this case, he may prefer to have children, as he prefers the large inheritance to go to his offspring rather than to a distant relative. In contrast, the incentive to deviate to a low fertility strategy is larger in the benchmark model, where generation 2 can appropriate the part of the bequest that would otherwise trickle down to the third generation. Furthermore, we show that in our model settlements are more likely to have a positive effect on fertility for discount functions that have a stronger degree of hyperbolicity (or "dynastic preference"). This suggests that hyperbolic discounting across generations is key to explain the reduced-form effect of settlements on fertility that we document in the empirical analysis.

Thirdly, we show that settlements emerged endogenously as an outcome of the family head's concerns over the survival of the dynasty and the heir's optimal decisions. We do so by showing that such commitment device is welfare improving for all the members of a dynasty with hyperbolic preferences. One the one hand, the family head is better off as settlements ensure the continuation of the family dynasty. On the other hand, the heir is *ex ante* better off. Under a settlement, he can credibly commit to have children, which guarantees that a larger share of the family wealth will trickle down from the family head. Since both the family head and his heir are better off, they may agree to sign a settlement as a result of their optimal decisions—even if this limits the heir's power to manage the family wealth.

Relative to the existing literature, we make the following contributions. First, our paper is the first to provide empirical evidence showing that inheritance schemes can change fertility incentives on the extensive margin, and hence, contribute to survival of family lineages. Second, while inheritance practices are usually treated as exogenous (Chu 1991), we propose a new theory where inheritance rules that restrict successors emerge as a result of the family head's concerns over the survival of the dynasty and the heir's optimal decisions. Altogether, these two contributions highlight the drawbacks of standard inheritance models that treat fertility as exogenous or ignore endogenous fertility decisions on the extensive margin—i.e., the decision to have children or not. Third, we show that classic models of bequests assuming exponential discounting (Barro 1974) can fail to explain inheritance schemes that restrict the successors' powers to manage inherited wealth (e.g., settlements or trusts). This is important, as such inheritance rules are widespread, especially among the very rich (Wolff and Gittleman 2014). Finally, we add to the large literature on inheritance systems by presenting settlements, which, despite receiving a lot of attention from contemporaries like Adam Smith, Tocqueville, or Karl Marx, are seldom considered by modern economists. We show that, as suggested by Adam Smith, settlements contributed to the perpetuation of elite lineages. Our results, however, suggest that they did so not only by entailing the land or favoring primogeniture, but also through changing fertility incentives. This challenges the common wisdom that fertility and inequality are negatively associated (Deaton and Paxson 1997; Kremer and Chen 2002; de la Croix and Doepke 2003). In contrast, our results suggest that an increase in the extensive margin of fertility can contribute to the survival of elites.

The article proceeds as follows. Section 2 reviews the literature. Section 3 describes settlements and the data. Section 4 presents reduced-form estimates on the effect of settlements on fertility. Section 5 provides robustness checks for the empirical results. In Section 6, we present our model of inheritance and bequests with hyperbolic discounting across generations. Finally, Section 7 concludes.

2 Related literature

This paper is related to several strands of the literature. The first is a rich literature showing that inheritance systems can have important effects for inequality or economic growth (Stiglitz 1969; Chu 1991; Piketty 2011; Bertocchi 2006). We add to this literature in two ways. First, we endogenize the emergence of inheritance systems. One common feature in the literature is to treat inheritance systems as exogenous. Notable exceptions are Chu (1991) and Grieco and Ziebarth (2015), who show that primogeneiture can emerge endogenously as a result of, respectively, concerns over the economic survival of the dynasty and insurance against income shocks. Differently, we show that inheritance practices can respond to fertility concerns. Second, most of the literature on inheritance treats fertility as exogenous or ignores endogenous decisions on the extensive margin—i.e., the decision to have children or not. In contrast, we show that inheritance schemes can affect this margin of fertility and, in turn, that concerns over childlessness can shape inheritance practices.

We also contribute to this large literature by studying settlements, an inheritance system that, despite receiving a lot of attention from contemporaries like Adam Smith, Tocqueville, or Karl Marx, is seldom considered by modern economists. The study of settlements has been focused on its functioning and has a descriptive nature (Habakkuk 1950; Bonfield 1979; English and Saville 1983).[4] We show that, as suggested by Adam Smith, settlements contributed to the perpetuation of elite lineages. Our results, however, suggest that they did so not only by entailing the land or favoring primogeniture, but also through changing fertility incentives.

Second, our paper contributes to a growing literature showing that the extensive margin of fertility (i.e., having children or not) can respond differently to economic changes than the intensive margin of fertility (i.e., the number of children). Aaronson, Lange, and Mazumder (2014) show that the Rosenwald Rural Schools Initiative decreased the number of children but increased motherhood rates. Similarly, Baudin, de la Croix, and Gobbi (2015) show that motherhood rates and completed fertility are negatively related for low-educated women. Brée and de la Croix (2016) show that materialism, women's empowerment, and the returns to education increased childlessness in Rouen (1640–1792). Finally, de la Croix, Schneider, and Weisdorf (2017) show that, once the extensive margin of fertility is accounted for, the lower-classes had higher reproductive success than the upper-classes in early-modern England. To the extent of our knowledge, this paper is the first to incorporate the dichotomy between the extensive and the intensive margin of fertility to the study of inheritance.

The third literature that motivates this paper studies time preferences. Hyperbolic discounting has been used to explain savings decisions (Laibson, Repetto, and Tobacman 1998; Diamond and Köszegi 2003), addictive behavior (Gruber and Köszegi 2001), or fertility (Wrede 2011; Wigniolle 2013) *of individuals*. We apply the idea of hyperbolic discounting *across generations*, in line with the seminal paper by Phelps and Pollak (1968). Our contribution is to show that models of bequests assuming

[4] The debate is focused on whether settlements were operational given that many family heads died before the wedding of their heirs, when settlements were typically signed.

exponential discounting are inconsistent with inheritance rules that restrict successors and that this type of discounting may ignore the effects of inheritance on the extensive margin of fertility.

Finally, this paper adds to our understanding of elites. Elsewhere it has been suggested that institutional capture (Acemoglu 2008; Allen 2009), primogeneiture (Bertocchi 2006), or marriage (Goñi 2018; Marcassa, Pouyet, and Trégouët 2017) consolidated elites in pre-modern Europe. We argue that settlements played a crucial role for the survival of the British aristocracy, as they reversed their astonishingly high childlessness rates in the 1600s. This result relates our findings to a literature studying the link between fertility and inequality in modern settings (Deaton and Paxson 1997; Kremer and Chen 2002; de la Croix and Doepke 2003). We argue that this relation may be different on the extensive and intensive margins of fertility.

3 Institutional Setting and Data

3.1 Settlements

How did settlements came into being? Before 1650, settlements were used exclusively to settle a provision for the wife in case she became a widow. Settlements were not used to entail the land because they were easy to break. A landowner who had settled his land could easily sell parts of the estate because nobody defended the interest of the beneficiary, that is, his under-aged or unborn son (Habakkuk 1994: p. 7). This changed during the interregnum period with the introduction of trustees, whose role was to defend the interest of under-aged or unborn beneficiaries.

Settlements developed during the Interregnum period for reasons unrelated to fertility concerns. After the Civil War, both Royalist and Parlamentarist landowners were afraid of expropriation in case events turned the tide in favor of the opposing side. Settlements ensured their family estates would not be lost. Note that when a landowner signed a settlement, the beneficiary of his estate was no longer him but his heir, most likely an under-aged kid or even an unborn son who had obviously not taken sides, and thus, who could not be expropriated (Habakkuk 1994: p. 12).

Although the threat of expropriation eventually disappeared, settlements became widely used by the aristocracy to entail the land and to fix a provision for wives and younger children. According to Habakkuk (1950), "about one-half of the land of

England was held under strict settlement in the mid-eighteenth century".

The typical settlement was signed upon the marriage of the eldest son. With the settlement, he limited his interest in the estate to that of a life-tenant, ensuring that the family estate would descend unbroken to the heir born of this marriage (Habakkuk 1950). In order to convince his son to make such a sacrifice, the father usually transferred him an income to support his household until he inherited the estate. Although settlements were only valid for a generation, *de facto* they operated as a permanent entailment of the land, as settlements were renewed by each generation. For settlements to operate in this fashion, however, it was crucial for the father to survive to the marriage of his eldest son (Bonfield 1979).

This demographic aspect of settlements is illustrated by the cases of the Brudenell and Craven families. Robert Brudenell, Earl of Cardigan, settled his estates in 1668 on the marriage of his eldest son. In contrast, the sixth Lord Craven died when his son was barely eighteen. As no settlement was signed, he could now sell parts of the family estate and break social rules by marrying at the advanced age of 37, a celebrated actress, Louisa Brunton (Habakkuk 1994: 19, 45, 46).

In the negotiation of settlements, the wife's family also had an interest, particularly on the allowances settled for her and for the younger children of the couple. Initially, such allowances were not prominent but by 1800 they became widespread (Habakkuk 1994: p. 16). Although we recognize the importance of allowances in the negotiation of settlements, we focus our analysis on settlements as a legal instrument to entail the land and ensure the integrity of family estates.

Importantly, settlements were prevalent in England, Wales, and Ireland, but not in Scotland. There, land could be entailed *ad perpetuum*. What frustrated the introduction of a similar form of permanent entailment in England is not clear. Habakkuk (1994: p. 18) suggests that the reasons may be purely legal and not related to any specific demographic aspect of these countries. Specifically, he suggests that the strong bias of English Common Law judges for the free alienability of land prevented the establishment of such permanent entails in England. In the empirical analysis, we exploit this divergence between England and Scotland to conduct placebo tests.

Settlements came to an end with the Settled Land Act in 1882. In the midst of a great debate about landownership concentration, Parliament established that settlements could not prevent the life tenant to sell parts of the land, as long as he obtained the best possible price and the profits from the sell were settled—that is, the money had to pass down untouched to the next generation (Habakkuk 1994: 1).

3.2 Data

We use genealogical data on the British peerage collected by Hollingsworth (1964). The dataset covers the entire period in which settlements were prevalent (1650-1882) and provides demographic information on c. 1,500 peer heirs and their wives. Unfortunately, the entries from the Hollingsworth (2001) dataset are not linked across generations. To resolve this, we manually matched each entry in the database to their father's entry. This subsection describes the original Hollingsworth (2001) dataset, the process of matching parents to offspring, and presents descriptive statistics.

The original data collected by Hollingsworth (1964) is from peerage records, which contain short biographical entries for all members of the aristocracy.[5] Hollingsworth (1964) tracked all peers who died between 1603 and 1938 (primary universe) and their offspring (secondary universe). In 2001, the Cambridge Group for the History of Population and Social Structure re-digitized the 30,000 original index sheets. In its current form, the data comprise c. 26,000 individuals. Each entry provides the date of birth, marriage, and death of each individual, as well as a variable indicating its accuracy. It also states the social status, title, whether he/she was heir-apparent at age 15, parent's status, and whether a title is an English, Scottish, or Irish peerage. Social status comprises five categories: (1) duke, earl, or marquis, (2) baron or viscount, (3) baronet, (4) knight, and (5) commoner. If the individual was married, we also know the spouses' date of birth, date of death, and social status. Each entry also lists the name and the date of birth of the children born to this marriage.

Unfortunately, the entries from the Hollingsworth (2001) dataset are not linked across generations. We manually matched each entry in the database to their parents entry. First, we match non-heirs (i.e., peers' daughters and younger sons) to their parents exploiting the fact that the reference number identifying non-heirs is a consecutive number of their father's reference number. The matching of heirs is less trivial: we match an entry C (children) to entry P (parent) if the information in entry C corresponds to what entry P reports about P's children. Specifically, we match entries according to the variables surname, name, date of birth, and accuracy. We perform four iterations in which matches are produced according to different combinations of

[5] For a list of all the peerage records used, see Hollingsworth (1964), appendix I.

these variables. At each iteration, we remove matched entries and we check double matches manually using information from thepeerage.com, an online genealogical survey of the peerage of Britain. We also use this webpage to double check matches in which the father's and children's surname display a Levenshtein distance above 1.[6] Finally, we try to match the remaining 1,503 unmatched heirs to their parents manually using information from thepeerage.com. Overall, we match 98.25 percent of the 26,499 entries in the dataset to their parents. Appendix A provides a detailed description of the matching process.

Table 1 presents descriptive statistics for c. 1,500 heirs to a peerage marrying between 1650 and 1882, and their wives. This is the main sample used in our empirical analysis. On average, 17 percent of married heirs remained childless. Admittedly, peers had children out of wedlock. Therefore, our childlessness rates might be an overestimate. However, illegitimate children did not inherit and therefore are not relevant for our analysis. Those who were not childless had, on average, 5.64 children. Wives were younger than husbands at marriage (22 versus 27 years old) and died at a similar age (60 versus 58 years). Around 50 percent of them had girls as the last child, indicating that on average parents did not stop having children after they had a son.[7] Regarding socio-economic status, 63 percent of the individuals were heirs to dukedom, an earldom, or a marquisate. Fourty-five percent are heirs to an English peerage, 31 percent to an Irish peerage, and 24 percent to a Scottish peerage, where settlements were not prevalent. In Section 4 we will exploit this sub-sample for a falsification test. Finally, 56 percent of the heirs married before their father's death, that is, they most likely signed a settlement. In the next section, we will use this proxy for settlements to gauge their impact on childlessness and completed fertility.

4 The effect of settlements on fertility

4.1 Historical trends

Compared to the general population, the British aristocracy had more children but a considerably higher childlessness rate. Figure 1 plots the average fertility of mothers

[6] The Levenshtein distance measures the minimum number of single-character edits required to change one surname into the other.

[7] For this variable, the sample is reduced to 899 because it considers heirs who had at least a child who (1) also appears in the Hollingsworth (2001) database and (2) who could be matched.

(left panel) and childlessness rates (right panel), for all peers' daughters first-marrying between ages 15 and 35 in 1600–1959. Dots illustrate the corresponding estimates for the general population.[8] On average, mothers in the aristocracy had between 4 and 5 children before the 1800s. The peerage experienced a demographic transition around 1810, eighty years earlier than the general population. This is consistent with previous findings regarding the fertility of the wealthy (Clark and Cummins 2009).

In contrast, marital childlessness rates among the aristocrats were astonishingly high. For example, around 1600 between 30 and 40 percent of all married women in the aristocracy were childless. In the general population, the corresponding rate was only c. 10 percent. The rate of childlessness in the peerage was high also in comparison to other European nobilities. For example, Pedlow (1982) and Lévy and Henry (1960) show that childlessness rates among the nobility of Hesse-Kassel (Germany) and of France were, respectively, 5 and 9 percent in 1650-99 (see Appendix B, Table 8).[9]

The high rates of childlessness in the peerage in 1600 were of course a threat for the continuity of noble family lineages. By 1650, however, childlessness rates started to decrease and by 1850 they reached 10 percent, the level for the general population (de la Croix, Schneider, and Weisdorf 2017). This declining trend coincides with the introduction of settlements. Next, we show that settlements crucially moved the peerage to a higher fertility regime, ensuring the perpetuation of noble lineages.

4.2 OLS estimates

Here we show that settlements reduced childlessness rates in the British peerage. Ideally, we would like to compare fertility outcomes in families that signed a settlement to the outcomes of similar families who did not signed it. Unfortunately, we do not know who signed a settlement and who did not. To resolve this issue, we exploit the demographic aspect of settlements. Most settlements were signed upon the marriage of the eldest son (Habakkuk 1950). In other words, for a settlement to be signed, it was crucial for the father to survive to the marriage of his eldest son (Bonfield 1979). In contrast, when the father died early, the heir would not be subject to a settlement;

[8] Estimates for the general population are from de la Croix, Schneider, and Weisdorf (2017), Galor (2011), Anderson (1998) and Wrigley et al. (1997).

[9] These comparisons have to be taken with grain of salt. First, Pedlow (1982) and Lévy and Henry (1960) base their estimates on a few observations. Second, the sample of French nobles is women marrying before 20, probably selecting women who married close relatives in pre-arranged marriages, which could affect childlessness rates (Goñi 2014).

he could dispose of the family estates at will, sell parts of it, and decide over the next generation's inheritance. We use the fact that a father survived (did not survive) until his heir's wedding as a proxy for the presence (abscence) of a settlement. Formally, we estimate the effect of settlements on childlessness as follows:

$$\chi_{i,j,b,q} = \beta \cdot S_i + \mu_j + \mu_b + \mu_q + \mathbf{X}'_{i,j,b,q}\gamma + \epsilon_{i,j,b,q} \quad (1)$$

where χ equals one if individual i did not have any children and equals zero otherwise. Our proxy for the presence of a settlement, S, indicates if i's father died after the wedding of his heir.[10] The coefficient β captures the association between settlements and childlessness. Following Galor and Klemp (2014), we include family fixed effects, μ_j, and cluster all standard errors by family. That is, we identify the effect of settlements on childlessness using variation in fertility among members of the same lineage. This will capture any genetic, cultural, religious, or socio-economic predisposition towards childlessness among these genetically related individuals. In addition, childlessness rates may be affected by the socio-economic and demographic conditions during one's lifetime. To capture such lifecycle effects, we include birth year fixed effects, μ_b, and dummies indicating the quarter-century in which the marriage took place, μ_q. Finally, the vector $\mathbf{X}$ includes a set of covariates that may also affect the probability of having children: social status of the spouses, wife's age at marriage, spouses' age at death, history of stillbirth in the husband's family, and the number of siblings of the husband. The latter accounts for the allowances for siblings, typically specified in the settlement.

Table 2 presents the results of estimating Equation (1) for all women marrying a peer or a peer heir between 1650 and 1882 using OLS.[11] There is a strong, significant association between the probability of being childless and settlements. Signing a settlement is associated with a decrease in the probability of being childless by 4 to 8 percentage points. Results are robust to the inclusion of covariates that may also affect childlessness, like the social status of spouses, the wife's age at marriage, or the

[10]Note that if i is the heir himself, then $S = 1$ when he married before his father's death. If i is not the heir of the family, then $S = 1$ when the family heir (i.e., i's eldest brother) married before the family head's death (i.e., before i's father's death).

[11]Our preferred specification is a linear probability model. The reason is that it is more flexible in dealing with fixed effects—which in our case are crucial to control for genetic, cultural, or religious unobserved factors affecting fertility at the family level. However, our baseline results are robust to using non-linear econometric models such as probit or logit (results are available upon request).

ratio of stillbirths to live births in the husbands family (cols. 2 and 3). The precision of the model increases when we include family fixed effects to control for unobserved heterogeneity in terms of genetic preconditions, culture, or social-economic position, as well as when we control for life-cycle conditions by including birth year and quarter-century of the marriage fixed effects (col. 4).

Is the total number of births—that is, the extensive margin of fertility—also associated to settlements? Historical trends suggest that, in contrast to childlessness, peers and commoners (who did not use settlements) present a comparable record for the number of births (see Figure 1). We should therefore expect settlements to play a minor role for the extensive margin of fertility beyond the effects over childlessness.

Table 2, column (5) confirms this. It presents results of poisson regressions[12] of Equation (1)'s form, with the number of births as dependent variable. To explain away the effect of settlements on childlessness, we restrict the sample to couples having at least one children. Results suggest that signing a settlement did not significantly affect the intensive margin of fertility: our proxy for settlements—i.e., marrying after the death of the family head—is not significantly associated with he number of live births, conditional on having at least one child. The estimates are small in magnitude: a coefficient of 0.036 indicates that an heir signing a settlement is expected to give birth to 3.6 percent more children than what he would have if he had not signed a settlement. Given that, conditional on not being childless, the average number of births of an heir's wife is 5.2, this effect is equivalent to having 0.19 more children.

Altogether, the evidence indicates a strong correlation between settlements and childlessness, but not with the number of births. In other words, settlements are associated with the extensive margin of fertility, while the effect on the intensive margin is negligible. Next, we use exogenous variation in our proxy for settlements to estimate the causal effect of this legal instrument on childlessness rates.

4.3 IV estimates

Here we estimate the causal effect of settlements on childlessness using an instrumental variables approach. Whether a family signed a settlement or not depends on many factors, some of which might be endogenous to childlessness. Specifically, it could be that individuals with certain characteristics that are correlated to childless-

[12]Poisson regressions are the standard form of regression analysis used to model count data like the number of live births.

ness may choose not to sign a settlement by delaying marriage until their father dies. We exploit exogenous variation in our proxy for settlements—i.e., the probability of marrying before or after the father's death—coming from the birth order of heirs.

The intuition of our instrument is very simple. Families who decide to have an heir cannot control the gender of the child. In some families, an heir might not be born until the second or third birth. Therefore, the father will be older, and thus more likely to die before his heirs wedding for exogenous reasons. In contrast, in families in which the first birth is a son, the father will be (exogenously) younger, more likely to survive until this sons wedding, and hence, more likely to sign a settlement.

Formally, we treat our proxy for settlements, S, as an endogenous variable:

$$S_i = \sum_{n=2}^{15} \beta_n \mathbb{I}(r_i = n) + \beta_z Z_i + \mu_q + \mathbf{X}'_{i,q}\gamma + \epsilon_{i,q} \ , \tag{2}$$

where S_i indicates if i's father survived until i's wedding.[13] That is, it is equal to one when i is likely subject to a settlement and equal to zero otherwise. Our principal instrument is $r_{i,q}$, the birth order of individual i. The indicator function $\mathbb{I}$ is equal to one when $r_i = n$ and zero otherwise. We also include the age at death of i's father, Z, which obviously affects S without regard to i's birth order. As in equation (1), μ_q are marriage quarter-century fixed effects; and $\mathbf{X}$ is a vector of covariates including social status of the spouses, wife's age at marriage, spouses' age at death, history of stillbirth in the husband's family, and the total number of siblings of the heir.

The causal effect of settlements on childlessness is captured by coefficient β in:

$$\chi_{i,j,b,d} = \beta \hat{S}_i + \mu_j + \mu_b + \mu_q + \mathbf{X}'_{i,j,b,q}\gamma + \epsilon_{i,j,b,q} \ . \tag{3}$$

where $\hat{S}_i$ is the value of S_i estimated from Equation (2), and μ_j and μ_b are, respectively, family and birth year fixed effects.

Note that our main specification is a triangular IV model in which not all the covariates used in the first-stage are included in the second-stage.[14] In detail, we include father's age at death in the first-stage but do not consider it to affect childlessness in the second-stage. The implicit assumption is that father's age at death

[13] For samples in which i is not the heir of the family, $S = 1$ when the family heir (i.e., i's eldest brother) married before the family head's death (i.e., before i's father's death).

[14] To fit this model, we estimate the recursive equation system (2) and (3) by maximum likelihood using the STATA user-written command cmp (Roodman 2015).

does not have a direct effect on childlessness other than affecting the probability of signing a settlement. This assumption would be violated, for example, if an early age at death of the father reflects poor health conditions that are transmitted intergenerationally. This scenario is unlikely for three reasons. First, we include the history of stillbirths in the second stage and estimate all the effects using family fixed effects. This captures any genetic predisposition towards childlessness. Conditional on these covariates, father's age at death likely does not affect childlessness. Second, we test the exogeneity of father's age at death formally by conducting Sargan-Hansen tests. Results suggest that, conditional on birth order being a valid instrument, father's age at death is exogenous to childlessness rates. Third, we present evidence suggesting that neither father's age at death nor birth order affect childlessness through channels other than settlements. In detail, we estimate the IV model for a comparable group of women who were not exposed to a settlement (because they married a non-heir or a Scottish peer) and show that the estimated effects are close to zero. Finally, the triangular structure of our IV model is not driving our results. In Appendix B we estimate a classic IV model (i.e., including all second-stage covariates in the first stage) and show that our main results are robust.

Table 3 presents the instrumental variables results. First-stage estimates show that, relative to first-born heirs, later-born heirs had a smaller probability to marry before their father's death and, hence, to sign a settlement. For example, a third-born heir was 10 percentage points less likely to sign a settlement, a fourth-born heir 11.9 percentage points, etc. The remaining covariates have expected signs. As the father lives longer, the probability of signing a settlement increases. The latter also increases with the husband's social status and decreases with wife's age at marriage. Finally, the F-test is large enough to eliminate concerns about weak instruments.

Second-stage estimates show that settlements had a negative, causal effect on childlessness. An heir marrying before his father's death and, thus, signing a settlement, was 14.7 percentage points less likely to be childless. The estimated effect is sizeable. Given that the average childlessness rate for heirs was 17.6%, settlements increased by 83.5% the extensive margin of fertility, pushing childlessness rates close to the "natural" rate of 2.4 percent (Tietze 1957). Note that the bias affecting the OLS results is an attenuation bias. A possible explanation is that if the father died before the wedding of the heir, he less likely influenced his son's choice of bride. In other words, the heir might have enjoyed more freedom when choosing his bride. If such marriages have more children (e.g., because they are love matches rather than socially convenient marriages), this could explain the attenuation bias in our OLS specification, corrected by the IV model.

Covariates have the expected signs. Specifically, marrying an older wife significantly increases the probability of not having children. However, note that the effect is much lower than that of settlements. In detail, to match the estimated effect of settlements on childlessness one would have to marry a wife aged 12 years younger.

Next, we present several empirical exercises to validate the identifying assumptions, that is, that the instrument is relevant and that the exclusion restriction is satisfied. Since we estimated a triangular IV model, we also need to show evidence for the validity of our triangular IV specification.

First stage results confirm that the birth order of the heir is a relevant instrument for our proxy for settlements: in families in which the heir is born after one or two daughters, the father is older and thus the likelihood that he survives until the heirs' wedding is smaller than if the heir is his first-born child. Furthermore, F-stats are large enough to rule out concerns about weak instruments.

Second, we examine the validity of the exclusion restriction, that is, that the heir's birth order affects childlessness only through the probability of signing a settlement. A potential concern is that birth order is associated with breast-feeding. In developing economies, it has been shown that breastfeeding increases with birth order, as mothers make use of the contraceptive properties of nursing when they hit the desired family size (Jayachandran and Kuziemko 2011). Since breast-feeding confers health benefits, low-birth-order heirs may be healthier, and hence, less likely to be childless. This scenario is unlikely in our historical context. Women in the aristocracy typically did not breastfeed their children; the common practice was to hire wet nurses (Fildes 1986: 193).[15] In other words, it is unlikely that breastfeeding is associated with birth order among aristocrats.

In addition, we can directly test for the exclusion restriction. Specifically, we exploit a unique feature of our historical setting to conduct placebo tests to validate our results. We estimate the instrumental variables system in Equations (2) and (3) with a comparable sample of women who should not be affected by settlements

[15]Moreover, the mechanism highlighted for developing countries is that women would like to limit the number of children because of budget constraints considerations. British Aristocrats were extremely wealthy and therefore did not face the same problem.

because (a) they did not marry an heir, or because (b) they married a Scottish heir. Unlike settlements in England and Ireland, Scottish entails were perpetual, i.e., they did not had to be renewed upon the heir's marriage (Habakkuk 1994: 6). If the exclusion restriction is satisfied—that is, if the birth order of the heir only affects childlessness through our proxy for settlements—we should find no effect for these populations.

Table 4 presents the results of these placebo tests of the exclusion restriction. The effect of marrying before the death of the family head (our proxy for settlements) on childlessness is much smaller and not significantly different from zero for non-heirs (col. 2).[16] In other words, for those who did not inherit the family estates, our proxy rightly indicates that settlements did not affect their choice of having children. A Wald test confirms that the estimated coefficients are significantly different from the baseline effect for the sample of heirs' wives (col. 1).

We find similar results when we compare women who married heirs to an English or Irish peerage (col. 3) to those that married heirs to a Scottish peerage (col. 4), and thus, who had not to renew entailments (the Scottish equivalent to a settlement) every generation. Signing a settlement decreases the probability of being childless by 16 percentage points in the case of women who married English or Irish heirs. In the case of wives of Scottish heirs the coefficient is not significantly different from zero. The Wald test rejects that the effect is the same for wives of English or Irish heirs and for wives of Scottish heirs. Note that, compared to the results in columns (1) and (2), the Wald test is weaker. This may be the result of the measurement error: on the one hand, there are fewer Scottish heirs, so the regression is estimated with fewer observations. On the other hand, Scottish peers typically held land and titles in England too, so some of them might have been subject to settlements.

Note that the Wald tests in columns (3) and (4) can be interpreted as difference-in-differences estimators. Specifically, the Wald test captures the differential effect of our proxy for signing a settlement on English and Irish heirs (treatment group) versus non-heirs or Scottish heirs (control group). In this difference-in-differences framework, results suggest that childlessness rates were reduced only for those who signed a settlement (i.e., heirs in England and Ireland who married before their father's death). In contrast, for non-heirs or heirs to a Scottish peerage, marrying before or after their

[16] Note that in this case the instrument is the birth order of the family heir, that is, the birth order of the husband's older brother.

father's death does not seem to affect childlessness.

Finally, the Sargan-Hansen test in Table 3 confirms the validity of our triangular IV specification. By ommitting father's age at death from the second stage we implicitly assumed that it does not have a direct effect on childlessness other than affecting the likelihood of signing a settlement. The test suggests that, conditional on birth order being a valid instrument, father's age at death is exogenous to childlessness.

Altogether, the evidence indicates that settlements had a negative, large causal effect on childlessness. Heirs born after several daughters were exogenusly less likely to marry before their father died. That is, they were exogenously less likely to sign a settlement, and thus, could freely dispose of the family estates, sell parts of it, and decide over the next generation's inheritance. As a result, their rates of childlessness were high. In contrast, first-born heirs were exogenously more likely to sign a settlement, which reduced their childlessness rates.

5 Robustness and extension

This section examines the robustness of our results and presents an extension of the analysis. First, we consider that settlements could be signed at the heir's majority instead of at the heir's wedding. Second, we estimate an alternative IV model exploiting variation in the gender of the first birth. Third, we explore whether the socioeconomic changes triggered by the Industrial Revolution affected our estimates.

5.1 Settlements signed at heir's majority.

So far, our empirical strategy assumes that a settlement was signed if the family head survived until the wedding of his heir. Although most settlements were signed upon marriage of the heir (Habakkuk 1994: 2), some settlements were signed when the heir turned 21, the age of majority. The reason was that

> the father might find it advantageous to bargain with his eldest son before a marriage was in immediate prospect to avoid the pressure of the bride's family. (Habakkuk 1994: p. 26).

Here we show that assuming that settlements were signed at the heir's majority does not alter our main conclusions. Formally, we estimate the IV model in equations 2 and 3 with an alternative proxy for settlements, S_i, indicating if i's father survived until i's majority. This alternative approach has the advantage of disentangling the two purposes of a settlement: entailing the land and setting a provision for the wife. As reflected in Habakkuk's quote, settlements signed at the heir's majority would only reflect the former, while settlements signed at the heir's wedding may also reflect the interest of the bride's family bargaining for a larger allowance.

Table 5 presents our main results using this alternative proxy for settlements. As before, we find that signing a settlement decreased the probability to be childless by 8 to 15 percentage points for women marrying a peer heir in 1650–1882. The magnitude of the IV coefficient (col. 2) is not significantly different to that of Table 3.[17]

The heir's birth order is also a relevant instrument under this alternative specification. First-stage results (Panel B) show that first-born heirs were more likely to turn 21 before their father's death than later-born heirs. Columns 3 to 5 present placebo tests of the exclusion restriction. The childlessness rates of women who were not exposed to settlements because they married a non-heir or a Scottish heir were not affected by the fact that the family head survived until his heir's majority or not. Wald tests reject the null hypothesis that the effects are the same for the sample of women marrying heirs and the sample of women marrying non-heir or Scottish heirs. In other words, our instruments do not seem to have a direct effect on childlessness in the placebo group. Finally, column 6 suggests that our alternative proxy for settlements is not associated with the intensive margin of fertility.

Finally, note that settlements signed at the heir's majority were not influenced by the interest of the bride's family as much as settlements signed at the marriage of the heir. The fact that we find similar results as before suggests that the effect of settlements on childlessness is the result of family interests to entail of land, and not the result of the bride's family interest in setting family provisions.

5.2 Alternative IV: gender of the first-born child.

Here we estimate the effect of settlements using an alternative instrument. We exploit exogenous variation in our proxy for settlements—i.e., the probability that an heir marries before his father's death—coming from the gender of the father's first child.

[17] As before, we find that the bias affecting the OLS results is an attenuation bias. Our previous conjecture for this was that if heirs delay marriage endogenously until their father's death they may also be more free to chose a bride and, hence, have higher fertility. This conjecture is also valid for the alternative proxy, as average age at marriage was 28.7, significantly above the age of majority.

Because one cannot manipulate children's gender, in some families the first-born will be a girl. There, the father will be older when his heir is born than what he would have been had his first-born been a boy. This generates an (exogenously) larger probability of dying before the heir's wedding, and hence, not signing a settlement.

Formally, we treat our proxy for settlements as an endogenous variable:

$$S_i = \beta_g G_i + \beta_z Z_{i,q} + \mu_q + \mathbf{X}_{i,q}^{'} \gamma + \epsilon_{i,q} \ , \tag{4}$$

where S_i indicates if i's father survived until the wedding of his heir. Our instrument is the gender of the first birth, G, which is equal to one when the first-born child of i's father was a daughter. As before, we include the age at death of i's father, Z; marriage quarter-century fixed effects, μ_q; and a vector of covariates $\mathbf{X}$ including spouses' social status and age at death, wife's age at marriage, the history of stillbirths in the husband's family, and the number of siblings of the heir. The second stage takes the same form as equation (3), where $\hat{S}$ is now estimated from equation (4).

This approach presents some advantages. In our main specification in Section 4, we used the birth order of the heir, that is, we exploit variation coming from the gender of *all* the births occuring before an heir is produced. This instrument is correlated with the size of the family. Potentially, this is problematic if, for example, larger families with a lot of daughters to marry off would become cash-constrained due to dowry payments. This scenario is unlikely—peerage families were extremely wealth (Rubinstein 1977). However, we use this alternative approach to fully rule-out such concerns, as the gender of the first-birth *alone* is not correlated with family size.

Table 6 (Panel B) presents the first-stage results. In families in which the first-born child was a girl, the heir was eight percentage points less likely to marry before his father's death, and hence, to sign a settlement. As before, the F-test is large enough to eliminate concerns about weak instruments. Second-stage results (Panel A) are also consistent with our previous findings. We find that signing a settlement decreased the probability to be childless by 14.6 percentage points. Columns 3 to 5 present the results of a placebo test where we run our alternative IV model for a sample of women who were not exposed to settlements because they married a non-heir (col. 3) or a Scottish heir (col. 5). Their childlessness rates were not affected by the fact that the family head survived until his heir's wedding. Wald tests reject the null hypothesis that the effects are the same for the sample of women marrying heirs and the placebo sample of women marrying non-heir or Scottish heirs.

5.3 Settlements before and after the Industrial Revolution.

A natural question is whether the effect of settlements varied over time. Specifically, our time window (1650–1882) includes the Industrial Revolution, an event that triggered major economic, demographic, and social changes. Whether this weakened the effect of settlements on fertility or not is an open question. On the one hand, the value of land relative to industrial wealth likely decreased after the Industrial Revolution.[18] Aristocrats might have faced lower incentives to sign a settlement, and hence, consolidate their family landholdings. This should reduce the strength of the effect of settlements on fertility. On the other hand, according to Doepke and Zilibotti (2008), the "fine tastes for leisure" of the landowners were not affected by the Industrial Revolution; they continued to live off their land rents. If this was the case, neither the incentives to sign a settlement nor its effects on fertility should be altered.

To answer this question, we split our sample between marriages occuring before and after the Industrial Revolution. The estimated effects remain stable. Table 7, col. (1) presents our baseline IV-estimates. Heirs who married before their father's death, that is, heirs who signed a settlement, were 14.8 percentage points less likely to be childless. In columns (2) and (3), we restrict the sample to matrimonies occuring before and after 1770—a date that marks the start of the first Industrial Revolution.[19] The estimated effects are very similar to those in the baseline specification.

Overall, this suggests that preferences of aristocrats over signing a settlement and over fertility persisted over time, even after the Industrial Revolution. This provides empirical support for the theory developed by Doepke and Zilibotti (2008), which claims that preferences (over leisure) of landowners were constant in time, eventually triggering their downfall as the economically dominant group.

[18]This is a relative statement. It was not until the twentieth century that industrial wealth became more important than landownership. For example, from 1800 to the 1870s, 80–95 percent of millionaires were still landowners (Rubinstein 1977: 102).

[19]We chose this year to mark the start of the Industrial Revolution because the 1770s saw the patenting of the spinning jenny (1770), the installation of a water frame in a cotton mill (1771), the formation of the Boulton & Watt partnership (1775), or the invention of the spinning mule (1779).

6 A model of inheritance and fertility

So far, we have shown that the inheritance practices of the peerage had a causal effect on the extensive margin of fertility—that is, the decision to have children or not. Next, we show that the extensive margin of fertility can shape inheritance rules. We build a model with inter-generational hyperbolic discounting where inheritance rules affect fertility and, in turn, schemes restricting successors (e.g., settlements or trusts) emerge endogenously in response to concerns over the dynasty's survival.

6.1 Setup

We assume a three-period sequential move game played by three generations, $i = \{1, 2, 3\}$, of the same dynasty. One can think of these as father, son, and grandson. Each generation makes decisions regarding consumption, x_i, and fertility, $n_i = \{0, 1\}$. We model fertility as a binary choice and assume that there is no uncertainty regarding having an heir.[20] If a generation decides not to have children, we assume that the dynasty dies out after this generation.[21]

Each dynasty is endowed with wealth K (e.g., landholdings). This endowment is used to subsidize consumption of all generations. Therefore, the decisions of each generation depend on how the dynasty wealth K is passed down from one generation to the next. This, in turn, depends on the degree of altruism towards future generations and on inheritance rules. We depart from the classic bequests models by assuming hyperbolic discounting towards future generations. This means that individuals are present biased but, at the same time, do not value their children's well-being significantly more than that of the future generations, namely their grandsons.

As for inheritance rules, we consider two models: a benchmark case in which each generation decides the bequests of the next generation, and a model with commitment in which the first generation decides the bequests of the following two generations. The latter is meant to represent settlements, which ensured the father some control over the inheritance that his grandson would receive. More generally, it represents any inheritance scheme that restricts successors' capacity to manage the family wealth.

[20] Alternatively, Li and Pantano (2014) model fertility choices in a dynamic framework in order to account for sex selection. In our setting, sex selection was not prevalent: 49% of last births were girls, suggesting that families did not stop having children after conceiving an heir (see Table 1).

[21] This could also be interpreted as the wealth passing down to a distant relative, whose utility is fully discounted. In Appendix D, we relax this assumption and show that results are robust.

6.2 Model without commitment

Consider a model in which each generation decides over consumption, fertility, and the bequest for the next generation. Generation 1 derives utility from his consumption and that of the following two generations in case the dynasty continues. The dynasty becomes extinct if any generation decides not to have children. Otherwise it dies out after generation 3.[22] Formally, the utility of generation 1 is

$$v_1(x_1, x_2, x_3, n_1, n_2) = u(x_1) + n_1 \cdot \left[\beta\delta\ u(x_2) + n_2 \cdot \beta\delta^2\ u(x_3)\right], \tag{5}$$

where x_i and $n_i = \{0, 1\}$ are the consumption and fertility of generation $i = \{1, 2, 3\}$.

We assume that generation 1 has a quasi-hyperbolic discount function towards future generations. This function has two components: First, $\delta \in [0, 1]$ is the standard discount for future generations. Second, $\beta \in [0, 1]$ discounts all the future consumptions compared to his own. This additional discount factor captures his dynastic preferences. Consider Figure 2: for low values of β, generation 1 has a strong dynastic preference, as he discounts his grandson and his son similarly. For high values of β, the discount function tends to the exponential discount function, implying that he values the consumption of his grandson much less than that of his son.

Hyperbolic discounting is important because it provides the rational for settlements, or more generally, inheritance rules that restrict successors. Under exponential discounting ($\beta = 1$) generation 1's preference for his own consumption x_1 relative to his son's consumption x_2 is no different from his son's preference for his own consumption x_2 relative to the succeeding generation's consumption x_3. In other words, preferences are time consistent across generations. This is hard to reconcile with any scheme that restrict successors. In contrast, under hyperbolic discounting ($\beta < 1$) individuals value the well-being of their grandchildren relatively more than their son will do, and hence, they have an incentive to restrict the son's capacity to manage the family wealth, for example, with a settlement.

Generation 1 is subject to a budget constraint. He allocates the family wealth,

[22] We abstact from parental investments in shaping children's preferences. Doepke and Zilibotti (2008) provide a model of endogenous preferences (without fertility decisions) showing that landowner's preferences are constant over time, even after the Industrial Revolution. Hence, our theoretical results should be robust to allowing parents to shape their children's preferences.

K, to his own consumption, and the bequest to the next generation, k_2. Formally,

$$K = x_1 + k_2. \tag{6}$$

Generation 2 faces a similar problem. He derives utility from his consumption, x_2, and from the consumption of generation 3 in case he decides to have children. His utility is therefore represented as

$$v_2(x_2, x_3, n_2) = u(x_2) + n_2 \cdot \beta\delta\ u(x_3). \tag{7}$$

He allocates the bequest he receives from his father, k_2, to his own consumption, and the bequest for the next generation. Formally, his budget constraint is

$$k_2 = x_2 + k_3. \tag{8}$$

Finally, generation 3—the grandson—faces a trivial problem. The dynasty will die out after him, so he only values his own consumption:

$$v_3 = u(x_3). \tag{9}$$

As there is no intrinsic utility from having children, he will consume all his endowment, that is, the bequests he receives from previous generations, k_3.

Since we have a sequential move game with perfect information and finite time, we use subgame perfect equilibrium (SPE) as the solution concept.

Definition 1 (SPE without commitment) *The SPE of the sequential game in which each generation decides over the bequests for the next generation is a strategy profile* $\{k_2, k_3, x_1, x_2, x_3, n_1, n_2, n_3\}$ *where* $\{k_2, x_1, n_1\}$ *maximize* v_1 *subject to (6),* $\{k_3, x_2, n_2\}$ *maximize* v_2 *subject to (8), and* $\{x_3, n_3\}$ *maximize* v_3 *subject to* $x_3 = k_3$.

We solve the model described in (5)-(9) by backward induction. Proposition 1 summarizes the optimal decisions regarding consumption and bequests conditional on fertility choices. Hereafter, we assume log-utility for simplicity; i.e. $u(x_i) = \ln x_i$.

Proposition 1 (Consumption and bequests without commitment) *Suppose each generation decides over the bequests for the next generation. In any SPE:*

(*a*) *If* n_1=0*, generation 1 consumes all the dynasty wealth,* $x_1 = K$.

(*b*) *If* $n_1{=}1$ *and* $n_2{=}0$*, generations 1 and 2 consume* $x_1^*{:=}\dfrac{K}{1+\beta\delta}$ *and* $x_2^*{:=}\dfrac{\beta\delta K}{1+\beta\delta}$ *respectively, and generation 1 gives a bequest* $k_2^* := x_2^*$.

(*c*) *If* $n_1{=}1$ *and* $n_2{=}1$*, generations 1, 2, and 3 consume* $x_1^{**} := \dfrac{K}{1+\beta\delta+\beta\delta^2}$, $x_2^{**} := \dfrac{1+\delta}{1+\beta\delta}\,\dfrac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$*, and* $x_3^{**} := \dfrac{\beta(1+\delta)}{1+\beta\delta}\,\dfrac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$ *respectively, and generations 1 and 2 give a bequest* $k_2^{**}{:=}K{-}x_1^{**}$ *and* $k_3^{**}{:=}x_3^{**}$ *respectively.*

Proof: See Appendix C.1. ■

Given these optimal consumption levels $K, x_1^*, x_2^*, x_1^{**}$, and x_2^{**}, we derive the optimal fertility choices for each generation by comparing the indirect utilities of having children and being childless. Proposition 2 characterizes these fertility decisions.

Proposition 2 (Fertility without commitment) *Suppose each generation decides over the bequests for the next generation. In any SPE:*

(*a*) *Generation 3 never has children,* $n_3 = 0$.

(*b*) *Generation 2 has children,* $n_2 = 1$*, if and only if:*

$$f(k_2, \beta, \delta) := v_2\left(x_2{=}\frac{k_2}{1+\beta\delta}, x_3{=}\frac{\beta\delta k_2}{1+\beta\delta}, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0) > 0,$$

where $f_{k_2} > 0$.

(*c*) *Generation 1 has children,* $n_1 = 1$*, if and only if:*

$$\begin{aligned} g(K, \beta, \delta) &:= v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) \\ &\quad - v_1(x_1{=}K, x_2{=}0, x_3{=}0, n_1{=}0, n_2{=}0) > 0 \quad \textit{when } f(k_2^*, \beta, \delta) < 0, \end{aligned}$$

or

$$\begin{aligned} h(K, \beta, \delta) &:= v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) \\ &\quad - v_1(x_1{=}K, x_2{=}0, x_3{=}0, n_1{=}0, n_2{=}0) > 0 \quad \textit{when } f(k_2^{**}, \beta, \delta) > 0 \end{aligned}$$

where $g_K, h_K > 0$.

Proof: See Appendix C.2. ■

This proposition argues that individuals will have children if and only if the indirect utility of doing so exceeds the indirect utility of being childless. For generation 2, the gains of having children are captured by the function f, where $x_2{=}\frac{k_2}{1+\beta\delta}$ and $k_3{=}x_3{=}\frac{\beta\delta k_2}{1+\beta\delta}$ are the optimal consumption and bequest decisions when he has children, and $x_2{=}k_2$ and $k_3{=}0$ are the corresponding optimal decisions when he is childless.[23] Importantly, f depends positively on his wealth, k_2. That is, the larger the bequest that generation 2 receives from his father, the more likely he is to have children.

For generation 1, the gains from having children depend on wealth but also on whether he anticipates to have a grandson or not. Specifically, function g captures the difference in indirect utilities from having children or not when the dynasty dies out after generation 2; i.e., when $f{<}0$. Function h captures the corresponding difference for the case in which the dynasty continues until generation 3; i.e., when $f{>}0$.

Proposition 3 describes the conditions for the three possible SPE: a high-fertility SPE in which generations 1 and 2 have children, a low-fertility SPE in which only generation 1 has children, and a no-fertility SPE in which generation 1 is childless.

Proposition 3 (SPE without commitment) *Suppose each generation decides over the bequests for the next generation. Then,*

(*i*) *A high-fertility strategy* $\{k_2^{**}, k_3^{**}, x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1, n_3{=}0\}$ *is the SPE if:*

- (a) $f(k_2^{**}, \beta, \delta) \geq 0$; $h(K, \beta, \delta) > 0$; *and*
- (b) $v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $f(k_2^{*}, \beta, \delta) < 0$ *and* $f(k_2^{**}, \beta, \delta) > 0$.

(*ii*) *A low-fertility strategy* $\{k_2^{*}, k_3{=}0, x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0, n_3{=}0\}$ *is the SPE if:*

- (a) $f(k_2^{*}, \beta, \delta) < 0$; $g(K, \beta, \delta) > 0$; *and*
- (b) $v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) \leq v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $f(k_2^{*}, \beta, \delta) < 0$ *and* $f(k_2^{**}, \beta, \delta) > 0$.

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}n_3{=}0\}$ *is the SPE if* $g(K, \beta, \delta) \leq 0$ *and* $h(K, \beta, \delta) \leq 0$.

Proof: See Appendix C.3. ■

For each SPE, condition (a) guarantees that generation 1 and generation 2 take the optimal fertility decisions for a given k_2, k_3, x_1, x_2, x_3. Condition (b) ensures that generation 1 internalizes optimally that he can influence the fertility choices of the second generation. Specifically, for some parameter values, both $f(k_2^{*}, \beta, \delta) < 0$ and

[23] To define f we characterize the indirect utility of being childless as $v_2(x_2 = k_2, x_3 = 0, n_2 = 0)$. This is a slight abuse of notation, as $n_2 = 0$ implies that the third generation does not exist. Hence, x_3 is not zero but undefined. Since the utility carried by future generations is multiplied by n_i this is inconsequential. The same applies for the indirect utility of being childless of generation 1.

$f(k_2^{**}, \beta, \delta) > 0$ hold. Generation 1 can then choose between an SPE in which he gives a low bequest k_2^* and generation 2 is childless and an SPE in which he gives a high bequest k_2^{**} and generation 2 has children. Condition (b) guarantees that generation 1 chooses his preferred SPE when these two are feasible.

The SPE crucially depends on intergenerational discounting. Figure 3 panel (a) shows how the two discount factors, β and δ, affect fertility choices for a given wealth K. Two comparative statics emerge: First, present-biased individuals are likely to pursue a low or a no-fertility strategy. Intuitively, for low values of β and δ, i.e., when individuals are present-biased, generation 1 consumes all the dynasty wealth and is childless. When individuals value future generations more, both generation 1 and 2 have children. For intermediate levels of β and δ, only generation 1 has children.

Second, for more hyperbolic discount functions a high-fertility strategy is the unique SPE. To see this we first need to define a measure capturing the degree of hyperbolism of the discount function. Remember that the discount function has two elements: the discount rate for future generations, δ, and the discount rate for all the future consumptions, β. On the one hand, for low values of β (and δ) individuals are present biased. On the other hand, for low values of β preferences are more hyperbolic; i.e., an individual does not value the consumption of his son significantly more than that of his grandson. To disentangle the two effects of β, we consider combinations of β and δ with the same degree of present-biasedness; i.e., we keep $\beta \cdot \delta$ constant. These combinations are represented by the isolines in Figure 3. Along a given isoline, lower values of β capture more hyperbolic discount functions.[24]

Definition 2 (Hyperbolic discounting) *A discount function defined by* $\{\beta, \delta\}$ *is more hyperbolic than a discount function defined by* $\{\beta', \delta'\}$ *if* $\beta \cdot \delta = \beta' \cdot \delta'$ *and* $\beta < \beta'$*.*

Figure 3 panel (a) suggests that dynasties with more hyperbolic discounting—that is, dynasties with a lower β along a given isoline—are more likely to be in a high-fertility regime. Intuitively, when the first generation does not value the consumption of his son significantly more than that of his grandson he has a higher incentive to keep the dynasty alive. Hence, he sets a bequest for generation 2 high enough to ensure a positive fertility. Proposition 4 generalizes this result.

[24]Formally, let $\beta \cdot \delta = \Gamma$. Generation 1 discounts the consumption of the next two generations with Γ and $\frac{\Gamma^2}{\beta}$ respectively, where $\beta \in [\Gamma, 1]$. Keeping Γ constant, a lower β is associated with a more similar dicounting for the next two generations; that is, a more hyperbolic discount function.

Proposition 4 (Comparative statics without commitment) *Suppose each generation decides over the bequests for the next generation. The conditions for a high-fertility SPE* $\{k_2^{**}, k_3^{**}, x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1, n_3{=}0\}$ *are more likely to be satisfied for more hyperbolic discount functions.*

Proof: See Appendix C.4. ■

In sum, present-biased individuals are less likely to have children. In contrast, individuals with a hyperbolic discount function prefer the dynasty to survive, and hence, give higher bequests to ensure that the next generation will have children. Next, we solve a model in which generation 1 settles all bequests and show that this commitment device can be used to increase the fertility of generation 2.

6.3 Model with commitment

Consider a model in which generation 1 sets all bequests for future generations, k_2 and k_3. This commitment device resembles the settlement, which ensured the family head (e.g., the father) control over the inheritance that the next generation's heir (e.g., the grandson) would receive. More generally, it represents any inheritance scheme restricting successors. As before, each generation decides over fertility and consumption. Hence, the constraints of generations 1 and 2 are now, respectively

$$K = x_1 + k_2 + k_3 \tag{10}$$

and

$$k_2 = x_2. \tag{11}$$

The decision problem of each generation is now characterized by equations (5), (7), (9), (10) and (11). Definition 3 characterizes the SPE of this game.

Definition 3 (SPE with commitment) *The SPE of the sequential game in which generation 1 decides over the bequests for the following two generations is a strategy profile* $\{k_2, k_3, x_1, x_2, x_3, n_1, n_2, n_3\}$ *where* $\{k_2, k_3, x_1, n_1\}$ *maximize* v_1 *subject to (10),* $\{x_2, n_2\}$ *maximize* v_2 *subject to (11), and* $\{x_3, n_3\}$ *maximize* v_3 *subject to* $x_3 = k_3$.

Proposition 5 shows the optimal consumption and bequest decision for this model.

Proposition 5 (Consumption and bequests with commitment) *Suppose generation 1 decides over the bequests for the following two generations. In any SPE:*

(*i*) *If* $n_1{=}0$*, generation 1 consumes all the dynasty wealth,* $x_1 = K$.

(ii) *If n_1=1 and n_2=0, generations 1 and 2 consume x_1^* and x_2^* and generation 1 gives a bequest $k_2 = x_2^*$ as in the model without commitment.*

(iii) *If n_1=1 and n_2=1, generation 1 consumes $x_{1c}^{**} := \dfrac{K}{1+\beta\delta+\beta\delta^2}$, generation 2 consumes $x_{2c}^{**} := \dfrac{\beta\delta K}{1+\beta\delta+\beta\delta^2}$, generation 3 consumes $x_{3c}^{**} := \dfrac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}$, and generation 1 chooses $k_{2c}^{**} := x_{2c}^{**}$ and $k_{3c}^{**} := x_{3c}^{**}$ as bequests.*

Proof: See Appendix C.5. ■

Proposition 5 suggests that the consumption and bequest decisions when n_1=0; and when both n_1=1 and n_2=0, are identical to the decisions from the model without commitment. In other words, the commitment device that allows generation 1 to set all bequests is only relevant when the dynasty does not die out, $n_1 = n_2 = 1$. In the latter case, note that $x_{2c}^{**} < x_2^{**}$ and $x_{3c}^{**} > x_3^{**}$. That is, generation 1 redistributes consumption from generation 2 to generation 3 by settling a larger bequest k_3 than the one generation 2 would have left in the model without commitment. Proposition 6 characterizes how this redistribution of family wealth affects fertility decisions.

Proposition 6 (Fertility with commitment) *Suppose that generation 1 decides over the bequests for the following two generations. In any SPE:*

(i) *Generation 3 never has children, $n_3 = 0$.*

(ii) *Generation 2 has children, $n_2 = 1$, if and only if:*

$$\mathcal{F}(k_3,\beta,\delta) := v_2(x_2{=}k_2, x_3{=}k_3, n_2{=}1) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0) > 0, \text{ where } \mathcal{F}_{k_3}>0$$

(iii) *Generation 1 has children, $n_1 = 1$, if and only if:*

$$g(K,\beta,\delta) := v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) - v_1(x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0) > 0$$

or

$$\mathcal{H}(K,\beta,\delta) := v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) - v_1(x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0) > 0 \text{ when } \mathcal{F}(k_{3c}^{**},\beta,\delta) > 0,$$

where $g_K, \mathcal{H}_K > 0$.

Proof: See Appendix C.6. ■

Compared to the previous model, the fertility choices for generation 2 change significantly. Here, the gains of having children are captured by the function $\mathcal{F}$, which no longer depends on generation 2's endowment, k_2, but on the endowment that generation 1 settled for generation 3, k_3. For example, when generation 1 sets k_3=0, generation 2 will always prefer to be childless. Hence, in the model with commitment generation 1 can influence the fertility choices of his son by settling more wealth for the third generation.

Proposition 7 describes the conditions for the three possible SPE: a high-fertility SPE in which generations 1 and 2 have children, a low-fertility SPE in which only generation 1 has children, and a no-fertility SPE in which generations 1 is childless.

Proposition 7 (SPE with commitment) *Suppose that generation 1 decides over the bequests for the following two generations. Then,*

(*i*) *A high-fertility strategy* $\{k_{2c}^{**}, k_{3c}^{**}, x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1, n_3{=}0\}$ *is the SPE if:*

  (*a*) $\mathcal{F}(k_{3c}^{**}, \beta, \delta) \geq 0$*;* $\mathcal{H}(K, \beta, \delta) > 0$*; and*

  (*b*) $v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$*,*

(*ii*) *A low-fertility strategy* $\{k_2^*, k_3{=}0, x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0, n_3{=}0\}$ *is the SPE if:*

  (*a*) $g(K, \beta, \delta) > 0$*, and*

  (*b*) $v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) \leq v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $\mathcal{F}(k_{3c}^{**}, \beta, \delta) > 0$*,*

(*iii*) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}n_3{=}0\}$ *is the SPE if* $g(K, \beta, \delta) \leq 0$ *and* $\mathcal{H}(K, \beta, \delta) \leq 0$*.*

Proof: See Appendix C.7. ■

For each SPE condition (a) guarantees that generation 1 and generation 2 take the optimal fertility decisions for a given k_2, k_3, x_1, x_2, x_3. Condition (b) ensures that generation 1 internalizes optimally that he can influence the fertility choices of generation 2. That is, that generation 1 chooses his preferred SPE when both the high-fertility SPE and the low-fertility SPE are feasible; i.e., when $\mathcal{F}(k_{3c}^{**}, \beta, \delta) > 0$.

As in the model without commitment, intergenerational discounting is crucial for fertility decisions. Specifically, present-biased individuals are more likely to be childless and more hyperbolic discount functions are associated with high-fertility strategies. Figure 3 panel (b) illustrates these two effects for a given endowment K. For low values of β and δ, that is, when individuals are present-biased, a no-fertility strategy is the unique SPE. In contrast, dynasties with more hyperbolic discounting are more likely to be in a high-fertility regime. As before, along a given isoline, the degree of present biasedness is constant and lower values of β are associated with a higher degree of hyperbolim. Dynasties with a lower β along a given isoline are more likely to be in a high-fertility regime. Intuitively, when the first generation has an hyperbolic discount function he does not value the consumption of his son significantly more than that of his grandson. Hence, he has a higher incentive to keep the dynasty alive. To achieve this, he settles a bequest for generation 3 high enough such that generation 2 prefers having children to being childless and loosing the utility from the settled bequest. Proposition 4 generalizes this result.

Proposition 8 (Comparative statics with commitment) *Suppose generation 1 decides over the bequests for the following two generations. The conditions for a high-fertility SPE* $\{k_{2c}^{**}, k_{3c}^{**}, x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1, n_3{=}0\}$ *are more likely to be satisfied for more hyperbolic discount functions.*

Proof: See Appendix C.8. ∎

6.4 Model comparison and main theoretical results

This subsection compares fertility choices across models and derives welfare implications. Three results emerge: we replicate our empirical result and identify the mechanisms behind it; we show that fertility concerns can endogenously shape inheritance practices; and we identify hyperbolic discounting across generations as the main rationale for inheritance schemes that restrict successors.

First, we show that commitment can increase fertility on the extensive margin; i.e., we show that the model for settlements replicates our empirical results. Note that, in both models, generation 1 prefers the dynasty not to die out when he is not present biased and when he has a hyperbolic discount function towards future generations (Propositions 4 and 8). This objective, however, is achieved differently in each model. In the model without commitment, generation 1 can increase the fertility of generation 2 by giving him a higher bequest k_2 (Proposition 2). In contrast, in the model with commitment generation 1 influences the fertility choice of generation 2 by settling a larger bequest k_3 for the third generation (Proposition 6).

The second mechanism is more effective in moving the dynasty to a high-fertility regime. For the sake of illustration, Figure 3 panel (c) compares equilibria across models. It plots the three different SPE of the game (no-fertility, low-fertility, and high-fertility) for different values of the discount factors β and δ and a given K. The highlighted area is the parameter region where commitment (strictly) increases fertility; i.e., where the second generation is childless in the model without commitment, but has children in the model with commitment. Proposition 9 generalizes this result.

Proposition 9 (The effect of settlements on fertility) *The set of parameter values that supports a high-fertility equilibrium in the model with commitment nests the corresponding set in the model without commitment.*

Proof: See Appendix C.9. ■

Intuitively, for any given bequest profile $\{k_2, k_3\}$, generation 2 has a lower incentive to deviate to a low-fertility strategy in the model with commitment than in the model without commitment. In the latter, generation 2 can choose to be childless and appropriate all the bequest k_2, which otherwise would be split between his own consumption and that of generation 3. In contrast, in the model with commitment, generation 2 cannot appropriate any of the bequest k_3 that generation 1 settled. If generation 2 deviates to a low fertility strategy, the dynasty dies out and k_3 is lost.[25] Hence, generation 1 can increase the fertility of generation 2 more effectively in the model with commitment (i.e., by settling a large bequest k_3) than in the model without commitment (i.e., by giving generation 2 a large bequest k_2).

This result highlights the importance of settlements (or any inheritance scheme that restricts successors) for fertility. Specifically, Proposition 9 reproduces our empirical finding: individuals who signed a settlement with their father were more likely to have children than individuals who were not subject to a settlement. As highlighted above, the economic intuition is simple: when an heir signs a settlement, he cannot sell parts of the family estate. He therefore has a higher incentive to have children, as he prefers the large, untouched inheritance to eventually go to his offspring rather than it to be lost (or to go to a distant relative).

Next, we show that hyperbolic discounting across generations can explain this reduced-form effect of settlements on childlessness. Note that commitment leads to a higher fertility only when discounting is hyperbolic. For example, figure 3 panel (c) shows that when β=1, that is, when discounting is exponential, the model with and without commitment produce identical fertility choices. Similarly, when individuals are highly present biased (low β and δ) or when they do not discount the future (high β and δ), fertility is identical across models. Only when the dynasty exhibits hyperbolic discounting, a high-fertility regime is more likely in the model with commitment. To

[25] Alternatively, one can think of k_3 going to a distant relative whose utilitty is fully discounted.

see this, note that lower values of β along a given isoline (which fixes $\beta{\cdot}\delta$ constant) lead to the parameter region where commitment is associated to high-fertility and no commitment to low fertility. Proposition 10 generalizes this result.

Proposition 10 (Settlements and hyperbolic discounting) *Fertility is larger in the model with commitment than in the model without commitment for more hyperbolic discount functions.*

Proof: See Appendix C.10. ■

Finally, we show that fertility concerns over the production of heirs and the survival of the dynasty can endogenously shape inheritance rules. To do so, Proposition 11 derives welfare implications. We compare each generation's utility across models in the parameter region where the model with commitment increases fertility.

Proposition 11 (Welfare) *In the parameter region where a high-fertility strategy is the SPE of the model with commitment and a low-fertility strategy is the SPE of the model without commitment, commitment is welfare improving. Specifically, all generations are better off; i.e.,* $v_3(x_{3c}^{**}) > v_3(x_3 = 0)$, $v_2(x_{2c}^{**}, x_{3c}^{**}, n_2{=}1) > v_2(x_2^{*}, x_3{=}0, n_2{=}0)$, *and* $v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0)$.

Proof: See Appendix C.11. ■

In other words, commitment is welfare improving for dynasties with hyperbolic discounting.[26] Generation 1 will always prefer the model with commitment, as this allows him to solve the problem of inter-generational time inconsistency: he can ensure the survival of the dynasty for two more generations and settle his preferred bequest to generation 3. In the model without commitment, instead, generation 2 would choose to pass down a smaller bequest or even let the dynasty die out, making generation 1 worse off. Importantly, generation 2 also prefers, *ex ante*, the model with commitment, even if this restricts his powers to manage the family wealth. The reason is that under such arrangement generation 2 can credibly commit to have children, which ensures that generation 1 will pass down a larger share of the family wealth K to the following two generations; i.e., $k_{2c}^{**} + k_{3c}^{**} > k_2^{*}$.[27]

This result shows that inheritance schemes that restrict successors (e.g., settlements) may emerge endogenously as an outcome of the family head's concerns over the survival of the dynasty and the heir's optimal decisions—even if this limits his

[26]From a societal point of view, however, the welfare implications are not clear.

[27]Obviously, generation 3 also prefers the model with commitment as otherwise he is not born.

powers to dispose of the family wealth.[28]

Finally, note that the parameter region where commitment is welfare improving corresponds to more hyperbolic discount functions. This highlights the importance of this type of discounting to rationalize the existence of settlements and, more generally, of inheritance rules that restrict successors. Furthermore, since the aristocracy and the very rich likely exhibit such dynastic preferences, this may explain the widespread use of settlements among aristocrats in the past or trusts among the wealthy today.

7 Conclusion

From 1650 to 1882, British peers could not freely dispose of their estates. Upon their marriage, peer heirs signed a settlement with their father in which they committed to pass down the family estate, unbroken, to the next generation. In this paper we show that such arrangements were crucial in reducing the high rates of childlessness in the British aristocracy, ensuring the survival of aristocratic family lineages. Using demographic evidence from about 1,500 heirs to a peerage and their wives between 1650 and 1882, we show that heirs marrying after their fathers' death—that is, heirs that were subject to a settlement—where 15 percentage points more likely to have children. To establish causality, we estimate an instrumental variables model that uses exogenous variation in the probability of signing a settlement coming from the birth order of the heir. In addition, we run placebo tests exploiting the fact that, in Scotland, entails had to be renewed upon the heir's marriage.

In the second part of the paper, we show theoretically that concerns over the production of heirs and the survival of a dynasty can endogenously shape inheritance practices. We depart from the classic bequests models by assuming that individuals have a quasi-hyperbolic discount function towards future generations. As preferences are not consistent across generations, fathers have an incentive to restrict their son's powers to manage the family estate with a settlement. We model settlements as a commitment device allowing the father to set all bequests for future generations. We show that the father can influence the fertility decisions of his son by settling a larger endowment for the third generation, namely the grandson. As a result, the dynasty

[28]Admittedly, according to this model signing a settlement should occur before the heirs marriage, as this reduces the probability that the father dies before the settlement is not signed. The reason why the signing of settlements was not anticipated is that settlements also included family provisions for the the bride and the younger children of the couple.

is less likely to die out than in a model where every generation decides the bequests of the next generation. This effect is stronger for more hyperbolic discount functions, showing that such time-preferences can rationalize the aforementioned reduced-form effect of settlements on fertility. Finally, we show that settlements emerged endogenously as an outcome of the family head's concerns over the survival of the dynasty and the heir's optimal decisions. We do so by showing that such inheritance practices can be welfare improving for all the members of a dynasty with hyperbolic preferences. One the one hand, the family head is better off as settlements ensure the continuation of the family dynasty. On the other hand, the heir is *ex ante* better off. Under a settlement, he can credibly commit to have children, which guarantees that a larger share of the family wealth will trickle down from the family head.

These results have three sets of implications: First, research on inheritance typically treats fertility as exogenous or ignores endogenous fertility choices on the extensive margin—i.e., to have children or not. In contrast, we show that inheritance schemes can affect this margin of fertility and, in turn, concerns over childlessness can determine inheritance practices. Second, we argue that models of bequests assuming exponential discounting (Barro 1974) are inconsistent with a inheritance rules which restrict successors' powers to manage the family wealth, and that this type of discounting may ignore important effects of such inheritance rules on fertility. This is important, as these inheritance practices were important in the past and, today, are widespread among the very rich (Wolff and Gittleman 2014). Finally, our results imply that, as suggested by Adam Smith, settlements contributed to the perpetuation of elite lineages. However, we argue that they did so not only by entailing the land or favoring primogeniture, but also through changing fertility incentives. This challenges the common wisdom that fertility and inequality to be negatively related.[29] This relation may be the opposite on the extensive margin of fertility.

References

Aaronson, Daniel, Fabian Lange, and Bhashkar Mazumder. 2014. "Fertility Transitions along the Extensive and Intensive Margins." *American Economic Review* 104 (1): 3701–3724.

[29] Deaton and Paxson (1997), Kremer and Chen (2002), de la Croix and Doepke (2003).

Acemoglu, Daron. 2008. "Oligarchic Versus Democratic Societies." *Journal of the European Economic Association* 6 (1): 1–44.

Allen, Douglas W. 2009. "A Theory of the Pre-Modern British Aristocracy." *Explorations in Economic History* 46 (3): pp. 299–313.

Anderson, Michael. 1998. "Highly Restricted Fertility: Very Small Families in the British Fertility Decline." *Population Studies* 52 (2): 177–199.

Atkinson, A. B., and A. J. Harrison. 1978. *Distribution of Total Wealth in Britain.* Cambridge University Press, Cambridge.

Barro, Robert J. 1974. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82 (6): 1095–1117.

Baudin, Thomas, David de la Croix, and Paula Gobbi. 2015. "Fertility and childlessness in the United States." *American Economic Review* 105 (6): 1852–1882.

Bertocchi, Graziella. 2006. "The Law of Primogeniture and the Transition from Landed Aristocracy to Industrial Democracy." *Journal of Economic Growth* 11 (1): 43–70.

Bonfield, Lloyd. 1979. "Marriage Settlements and the 'Rise of Great Estates': The Demographic Aspect." *The Economic History Review* 32 (4): 483–493.

Brée, Sandra, and David de la Croix. 2016. "Key Forces Behind the Decline of Fertility: Lessons from Childlessness in Rouen before the Industrial Revolution." Unpublished.

Chu, C. Y. Cyrus. 1991. "Primogeniture." *Journal of Political Economy* 99 (1): 78–99.

Clark, Gregory, and Neil Cummins. 2009. "Urbanization, Mortality, and Fertility in Malthusian England." *American Economic Review* 99 (2): 242–47.

Deaton, Angus, and Christina Paxson. 1997. "The effects of economic and population growth on national saving and inequality." *Demography* 34 (1): 97–114.

de la Croix, David, and Matthias Doepke. 2003. "Inequality and Growth: Why Differential Fertility Matters." *American Economic Review* 93 (4): 1091–1113.

de la Croix, David, Eric Schneider, and Jacob Weisdorf. 2017. "Decessit sine prole - Childlessness, Celibacy, and Survival of the Richest." Unpublished.

Diamond, P., and B. Köszegi. 2003. "Quasi-hyperbolic discounting and retirement." *Journal of Public Economics* 87 (9-10): 1839–72.

Doepke, Matthias, and Fabrizio Zilibotti. 2008. "Occupational Choice and the Spirit of Capitalism." *The Quarterly Journal of Economics* 123 (2): 747–793.

English, B., and J. Saville. 1983. *Strict Settlement: A Guide for Historians.* Hull: University of Hull Press.

Fildes, Valerie A. 1986. *Breasts, Bottles, and Babies: A History of Infant Feeding.* Edinburgh: Edinburgh University Press.

Galor, Oded. 2011. *Unified Growth Theory.* Princeton: Princeton University Press.

Galor, Oded, and Marc Klemp. 2014. "The Biocultural Origins of Human Capital Formation." Working paper 20474, National Bureau of Economic Research.

Goñi, Marc. 2014. "Kissing cousins. Estimating the causal effect of consanguinity on fertility using evidence from the London Season." Unpublished.

———. 2018. "Assortative Matching at the top of the Distribution. Evidence from the world's most exclusive marriage market." Unpublished.

Grieco, Paul L. E., and Nicolas L. Ziebarth. 2015. "Unigeniture in an Uncertain World." *Cliometrica* 9 (2): 139–166.

Gruber, J., and B. Köszegi. 2001. "Is addiction 'rational'? Theory and evidence." *Quarterly Journal of Economics* 116 (4): 1261–304.

Habakkuk, H. J. 1950. "Marriage Settlements in the Eighteenth Century." *Transactions of the Royal Historical Society.*

———. 1994. *Marriage, Debt, and the Estates System: English Landownership, 1650-1950.* Oxford: Clarendon Press.

Hollingsworth, T. H. 1964. "The Demography of the British Peerage." *Population Studies*, vol. Supplement.

———. 2001. "Hollingsworth Genealogical Data on the British Peerage". Database. Re-digitised from handwritten original index sheets and archived by the Cambridge Group for the History of Population and Social Structure.

Jayachandran, Seema, and Ilyana Kuziemko. 2011. "Why Do Mothers Breastfeed Girls Less than Boys? Evidence and Implications for Child Health in India*." *The Quarterly Journal of Economics* 126 (3): 1485–1538.

Kremer, Michael, and Daniel L. Chen. 2002. "Income Distribution Dynamics with Endogenous Fertility." *Journal of Economic Growth* 7 (3): 227–258.

Laibson, D. I., A. Repetto, and J. Tobacman. 1998. "Self control and saving for retirement." *Brookings Papers on Economic Activity* 1 (1): 91–172.

Lévy, Claude, and Louis Henry. 1960. "Ducs et pairs sous l'Ancien Régime. Caractéristiques démographiques d'une caste." *Population (French)* 15 (5): 807–830.

Li, Qi, and Juan Pantano. 2014. "The demographic consequences of sex-selectiono technology." *Unpublished*.

Marcassa, Stefania, Jérôme Pouyet, and Thomas Trégouët. 2017. "Marriage Strategy Among European Nobility." Unpublished.

Pedlow, Gregory W. 1982. "Marriage, Family Size, and Inheritance Among Hessian Nobles, 1650-1900." *Journal of Family History* 7 (4): 333–352.

Phelps, E. S., and R. A. Pollak. 1968. "On Second-Best National Saving and Game-Equilibrium Growth." *The Review of Economic Studies* 35 (2): 185–199.

Piketty, Thomas. 2011. "On the Long-Run Evolution of Inheritance: France 1820–2050." *The Quarterly Journal of Economics* 126 (3): 1071.

Roodman, David. 2015. CMP: Stata module to implement conditional (recursive) mixed process estimator. EconPapers.repec.org/RePEc:boc:bocode:s456882.

Rubinstein, William D. 1977. "Wealth, Elites and the Class Structure of Modern Britain." *Past and Present*, no. 76:99–126.

Smith, Adam. 1776. *An Inquiry into the Nature and Causes of the Wealth of Nations.* Reprint. New York: Modern Library, 1937.

Stiglitz, Joseph E. 1969. "Distribution of Income and Wealth Among Individuals." *Econometrica* 37 (3): 382–397.

Tietze, Christopher. 1957. "Reproductive Span and Rate of Reproduction Among Hutterite Women." *Fertility and Sterility* 8 (1): 89–97.

Wigniolle, Bertrand. 2013. "Fertility in the absence of self-control." *Mathematical Social Sciences* 66 (1): 71–86.

Wolff, Edward N., and Maury Gittleman. 2014. "Inheritances and the distribution of wealth or whatever happened to the great inheritance boom?" *The Journal of Economic Inequality* 12 (4): 439–468.

Wrede, M. 2011. "Hyperbolic discounting and fertility." *Journal of Population Economics* 24 (3): 1053–1070.

Wrigley, E.A., R. S. Davies, J.E. Oeppen, and R.S. Schoeld. 1997. *English Population History from Family Reconstitution: 1580-1837.* Cambridge: Cambridge University Press.

8 Figures and Tables

Figure 1: Childlessness rates and average births of mothers, by marriage decade.

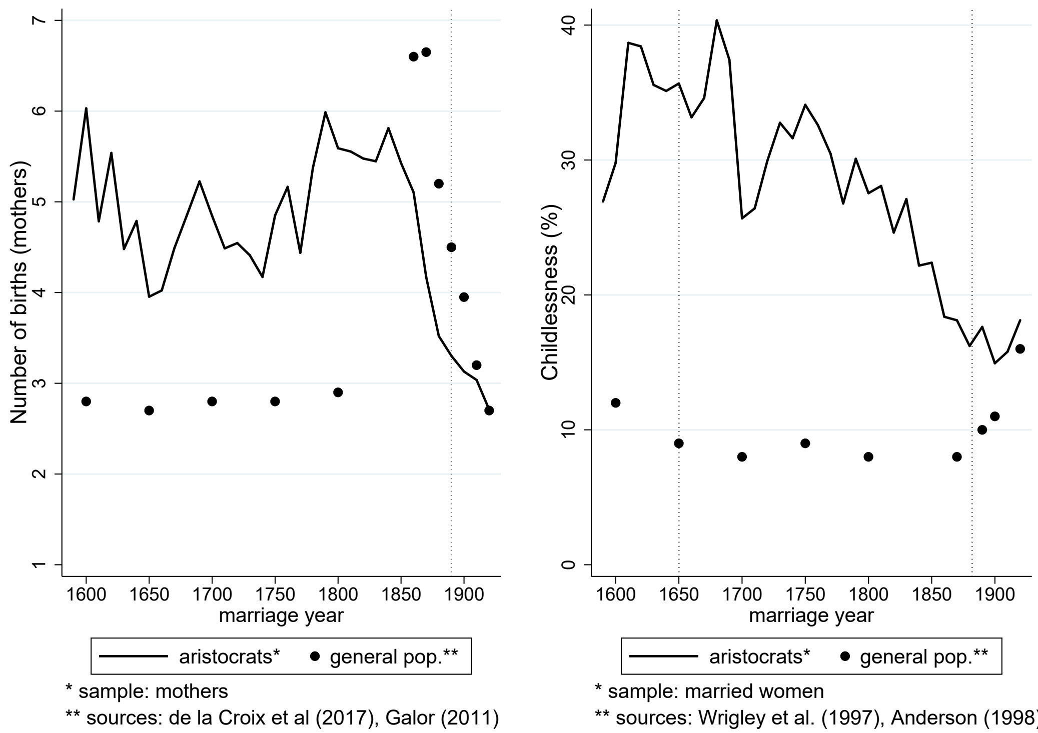


Figure 2: Quasi-hyperbolic discrete discount function

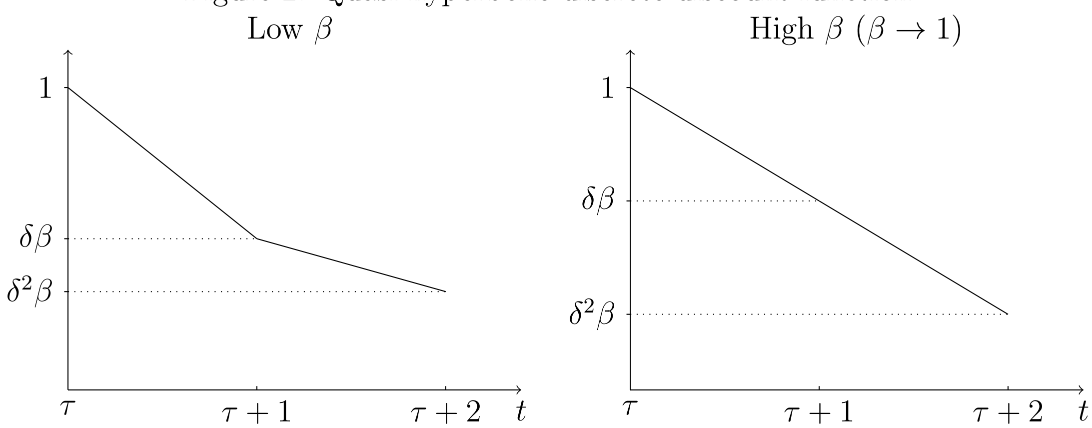
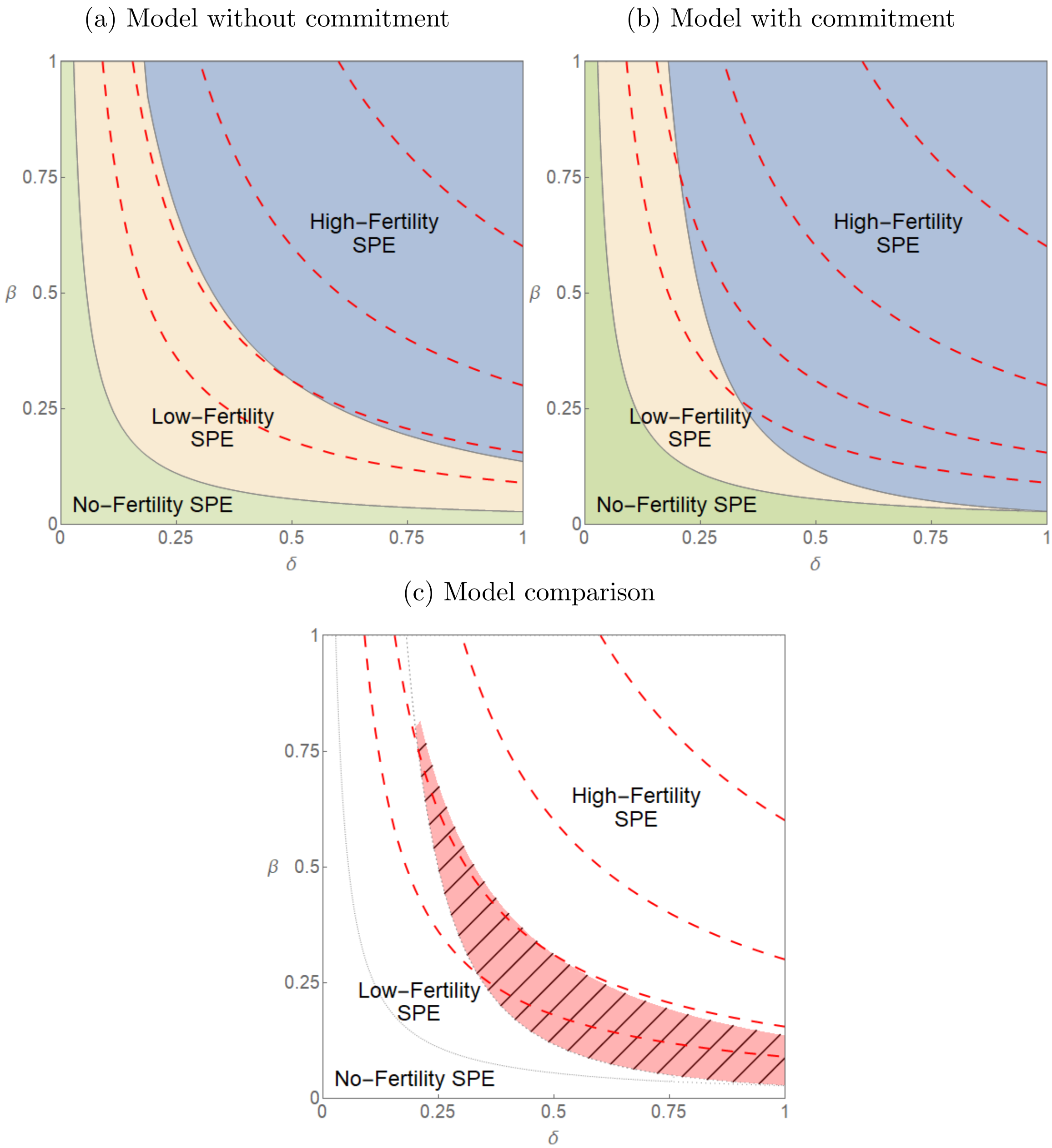


Figure 3: Discount factors and fertility



Region where commitment increases fertility (panel (c))

Isolines for $\beta \cdot \delta$ constant

Notes: Family wealth K is fixed to 100.

Table 1: Summary statistics for the Hollingsworth's dataset (1650–1882)

	mean	std. dev.	min	max	N
A. Fertility variables					
% childless	0.17	0.38	0	1	1,529
All live births	4.67	3.88	0	22	1,529
All live births (if > 0)	5.64	3.56	1	22	1,267
Stillbirths	0.24	0.73	0	9	276
B. Other demographic variables					
Age at first marriage (wife)	21.94	4.93	11	55	1,556
Age at first marriage (husband)	27.20	6.90	8	62	1,558
Age at death (wife)	58.37	20.22	16	100	1,553
Age at death (husband)	60.25	16.94	16	97	1,559
Age difference	-5.25	6.49	-35	23	1,556
Number of marriages	1.25	0.51	1	4	1,559
Last child is a girl	0.53	0.50	0	1	899
C. Socioeconomic status variables					
Baron heir	0.37	0.48	0	1	1,559
Duke heir	0.63	0.48	0	1	1,559
Wife is a commoner	0.58	0.49	0	1	1,559
English peerage	0.45	0.50	0	1	1,559
Scottish peerage	0.24	0.43	0	1	1,559
Irish peerage	0.31	0.46	0	1	1,559
Proxy for settlement [i.e., father died after wedding]	0.56	0.50	0	1	1,559

Notes: The sample are all marriages in 1650–1882 where the husband was heir to a peerage. Marriages to women below 12 are excluded (birth or marriage date was probably missreported).

Table 2: Baseline results

	(1)	(2)	(3)	(4)	(5)
			Childlessness		All live births (if > 0)
	OLS	OLS	OLS	OLS	poisson
Settlement	-0.050***	-0.052***	-0.036**	-0.079**	0.036
[i.e., father died after wedding]	(0.019)	(0.019)	(0.018)	(0.035)	(0.042)
Husband's siblings (#)	-0.001	-0.001	-0.001	-0.004	-0.010
	(0.002)	(0.002)	(0.002)	(0.005)	(0.006)
Duke heir		0.022	0.022	-0.041	0.042
		(0.019)	(0.018)	(0.049)	(0.076)
Baron heir		ref.	ref.	ref.	ref.
Wife's age at marriage			0.014***	0.014***	-0.024***
			(0.002)	(0.004)	(0.005)
Wife's age at death			0.000	-0.000	0.003***
			(0.000)	(0.001)	(0.001)
Husband's age at death			-0.003***	-0.004***	0.013***
			(0.001)	(0.001)	(0.002)
Still to live births (fam)			0.175	0.050	3.4
			(0.311)	(2.940)	(2.7)
Wife's social status	NO	YES	YES	YES	YES
Family FE	NO	NO	NO	YES	YES
Birth year FE	NO	NO	NO	YES	YES
Marr. quarter-century FE	NO	NO	NO	YES	YES
Observations	1,526	1,525	1,505	1,505	1,261
% correctly predicted	81.2	81.2	82.8	90.9	-

Notes: The sample are all marriages in 1650–1882 where the husband was heir to a peerage. In column (5), the sample is restricted to women who gave birth at least once. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 3: Instrumental variables' results

Second stage	Dep. Var.: Childlessness	
	coef.	s.e.
Settlement [i.e., father died after wedding]	-0.146***	(0.036)
Controls	YES	
Family and birth year FE	YES	
Marr. quarter-century FE	YES	
Observations	1,505	
% correctly predicted	91.1	

First stage	Settlement [i.e., father died after wedding]	
	coef.	s.e.
Birth order of the heir		
1st	reference	
2nd	-0.037	(0.024)
3rd	-0.102***	(0.026)
4th	-0.119***	(0.033)
5th	-0.118***	(0.045)
6th	-0.150***	(0.055)
7th	-0.165**	(0.074)
8th	-0.117	(0.106)
9th	-0.154	(0.114)
10th	-0.042	(0.093)
11th	0.108	(0.235)
12th	-0.139	(0.115)
13th	0.222	(0.196)
15th	0.426***	(0.049)
Father age at death	0.021***	(0.001)
Controls	YES	
Marr. quarter-century FE	YES	
Observations	1,530	
% correctly predicted	74.8	
F-test	110.0	
Sargan-Hansen test	13.12	p-val=0.4

Notes: The sample are all marriages in 1650–1882 where the husband was heir to a peerage. Controls: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in husband's family, spouses' social status; s.e. clustered by family; *** p<0.01, ** p<0.05, * p<0.1.

Table 4: Placebo test for the exclusion restriction

	(1)	(2)	(3)	(4)
Dep. Variable: Childlessness				
	benchmark IV	non-heirs IV	England and Ireland IV	Scotland IV
Settlement	-0.146***	0.031	-0.159***	0.025
[i.e., father died after wedding]	(0.036)	(0.054)	(0.054)	(0.093)
Ho:	-	$\beta(1) = \beta(2)$	-	$\beta(3) = \beta(4)$
prob > chi2	-	0.006***	-	0.087*
Controls	YES	YES	YES	YES
Family FE	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES
Father-in-law status	-	YES	-	-
Observations	1,506	1,442	1,139	366
% correctly predicted	91	54	79	40
F-stat from first stage	110	90	85	51

Notes: The sample are all marriages in 1650–1882 where the husband is heir to a peerage (col. 1), the husband is not a heir and the wife is a peers' daughter (col. 2), the husband is heir to an English or Irish peerage (col. 3), and the husband is heir to a Scottish peerage (col. 4). Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Robustness: settlements signed at heir's majority

	(1)	(2)	(3)	(4)	(5)	(6)
	heir	heir	non-heir	England and Ireland	Scotland	heir
Panel A: Second stage			Childlessness			All live births (if > 0)
	OLS	IV	IV	IV	IV	poisson
Settlement	-0.078***	-0.149***	0.031	-0.180***	0.033	0.018
[i.e., father died after heir's majority]	(0.030)	(0.038)	(0.054)	(0.055)	(0.053)	(0.040)
Ho:	-	-	$\beta(2) = \beta(3)$	-	$\beta(4) = \beta(5)$	-
prob > chi2	-	-	0.008***	-	0.005***	-
Controls	YES	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES	YES
Observations	1,699	1,699	1,807	1,264	434	1,415
% correctly predicted	90	90	58	77	33	-
Panel B: First stage	Dep. Variable: Settlement [i.e., father died after heir's majority]					
Birth order of the heir						
1st	-	reference	reference	reference	reference	-
2nd	-	-0.040**	-0.068**	-0.033	-0.070*	-
		(0.020)	(0.028)	(0.023)	(0.039)	-
3rd	-	-0.089***	-0.089**	-0.076**	-0.142***	-
		(0.025)	(0.038)	(0.030)	(0.048)	-
4th	-	-0.113***	-0.130***	-0.085***	-0.215***	-
		(0.026)	(0.040)	(0.028)	(0.063)	-
5th to 15th not reported						
Controls	-	YES	YES	YES	YES	-
M. quarter-century FE	-	YES	YES	YES	YES	-
Observations	-	1,699	1,807	1,264	434	-
F-stat	-	105.8	101.7	88	52.8	-

Notes: The sample are all marriages in 1650–1882 where the husband is heir to a peerage (cols. 1, 2, and 6), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). In col. (6), the sample is restricted to women who gave birth at least once. Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage also includes father's age at death as a covariate. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 6: Robustness: IV using the gender of the first birth

	(1)	(2)	(3)	(4)	(5)
				England	
	heir	heir	non-heir	and Ireland	Scotland
Panel A: Second stage	Dep. Variable: Childlessness				
Settlement	-0.146***	-0.146***	0.011	-0.176***	0.027
[i.e., father died after wedding]	(0.036)	(0.035)	(0.058)	(0.051)	(0.079)
Ho:	-		$\beta(2) = \beta(3)$	-	$\beta(3) = \beta(4)$
prob > chi2	-		0.022**		0.029**
Controls	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Observations	1,506	1,506	1,442	1,139	366
% correctly predicted	91	91	54	79	39
Panel B: First stage	Dep. Variable: Settlement [i.e., father died after wedding]				
Gender of first birth:					
son	-	reference	reference	reference	reference
daughter	-	-0.079***	-0.072***	-0.057**	-0.138***
		(0.020)	(0.019)	(0.022)	(0.041)
Instrument	birth order	daughters	daughters	daughters	daughters
Controls	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Observations	1,506	1,506	1,442	1,139	366
F-stat	110	160	80	122	73

Notes: The sample are all marriages in 1650–1882 where the husband is heir to a peerage (cols. 1 & 2), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage also includes father's age at death as a covariate. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 7: Extensions: the effect before and after the Industrial Revolution

	(1)	(2)	(3)
Dep. Variable: Childlessness			
	benchmark (1650–1882)	before IR (1650–1769)	after IR (1770–1882)
	IV	IV	IV
Settlement	-0.148***	-0.140**	-0.147**
[i.e., father died after wedding]	(0.036)	(0.059)	(0.064)
Controls	YES	YES	YES
Family FE	YES	YES	YES
Birth year FE	YES	YES	YES
Marriage decade FE	YES	YES	YES
Observations	1,530	708	823
% correctly predicted	91	94	94
Instrument	birth order	birth order	birth order
F-stat from first-stage	111	60	87

Notes: The sample are all marriages in the indicated years, where the husband was heir to a peerage. Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and spouses' social status. First-stage results not reported. Standard errors clustered by family in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Online Appendix

A Data appendix

This appendix describes in detail the process of matching parents to offspring in the Hollingsworth (2001) dataset.

To guide the reader, we first describe how the entries in the Hollingsworth (2001) dataset look like. Figure 4 shows the entry for James Hamilton, first Earl Abercorn. Each entry is identified by a reference number, in this case, zero. The entry reports James Hamilton's full name, surname, the date of birth, marriage, and death, as well as a variable indicating its accuracy. Importantly for our matching algorithm, the entry also lists the name and the date of birth of the children born to his marriage. In this case, James Hamilton had 9 children, two of which eventually inherited titles (James, 2nd Earl Abercorn and Claude, 2nd Baron Strabane).

Unfortunately, the entries from the Hollingsworth (2001) dataset are not linked across generations. In other words, there is no reference number that links this entry of James Hamilton, first Earl Abercorn, to the entry of his son James Hamilton, 2nd Earl Abercorn. To resolve this issue, we manually matched each entry in the database to their father's entry. For individuals whose father could not be found in the database we tried to match them with their mothers.

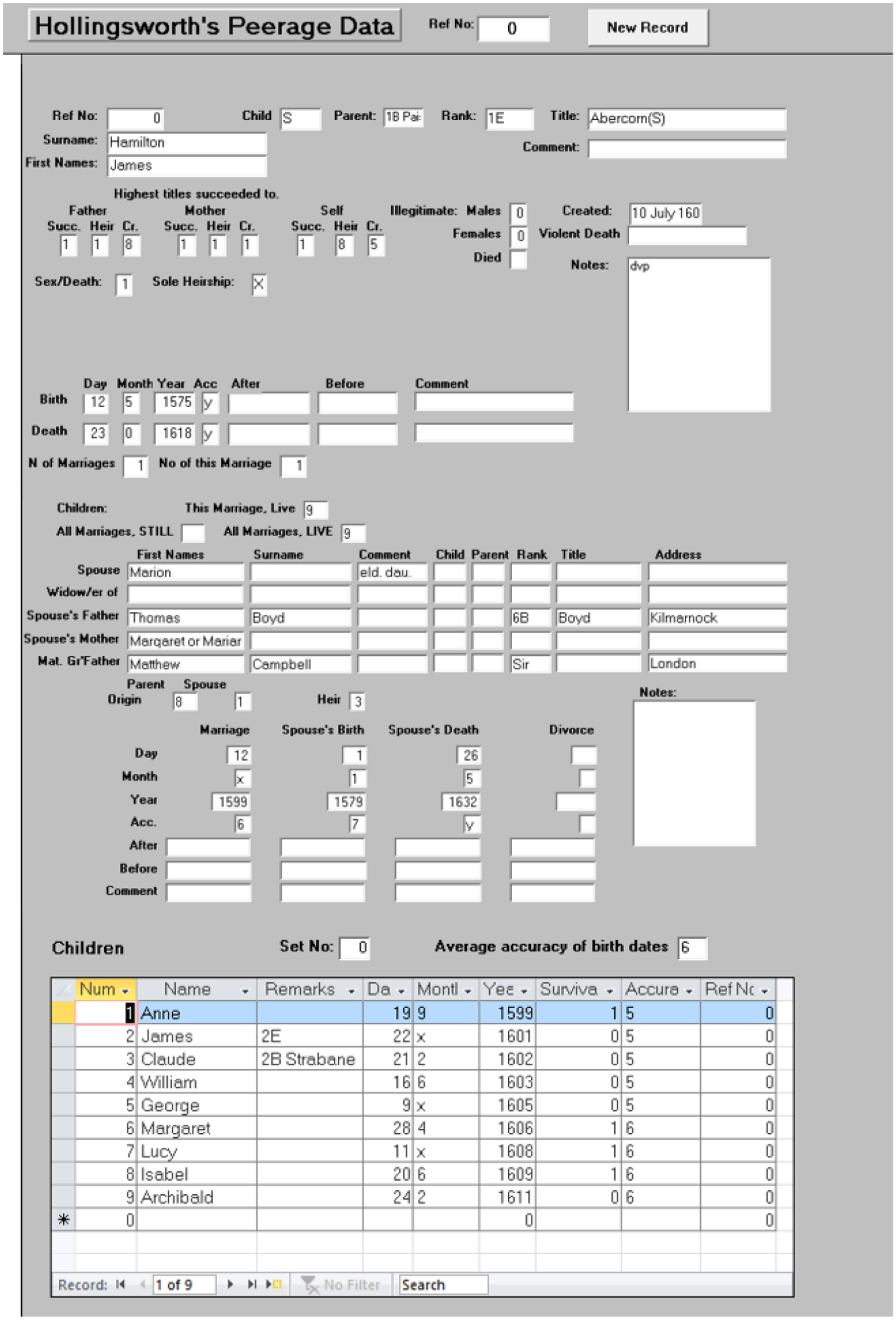
In detail, we first match non-heirs (i.e., peers' daughters and younger sons) to their parents. To do so, we exploit a particularity of the Hollingsworth (2001) database. An entry corresponding to a peer or a peer heir has a reference number which is typically a multiple of 20 or 50. The reference number for his daughters and younger sons (if any) are consecutive numbers of this (i.e., the father's) reference number. Thus, we match an entry C (children) to entry P (parent) if entry P has a reference number that is a multiple of 20 or 50 and entry C has a consecutive reference number. Using this procedure, we match 12,593 peers' daughters and 9,240 peers' younger sons to their parents.

The matching of heirs is less trivial. It involves four iterations. In the first iteration, we match entries C and P if entry P corresponds to a male and the information in entry C corresponds to what entry P reports about P's children. Specifically, we match entries C and P if the C's surname, name, date of birth, and accuracy coincides with P's surname and the name, date of birth, and accuracy of any of the children listed in entry P. We then restrict the sample to unmatched individuals, and repeat the procedure considering female P entries only. This concludes iteration 1. For the remaining unmatched individuals, we consider a similar matching procedure based on birth date and accuracy (iteration 2), first name and birth date (iteration 3), and unique birth dates—that is, restricting the sample to individuals born on a date where no other peer or peer's offspring was born (iteration 4). At each iteration, we check double matches manually using information from thepeerage.com, an online genealogical survey of the peerage of Britain. Finally, we try to match the remaining unmatched heirs to their parents using information from thepeerage.com. Using this iterative procedure, we match 4,666 peers' heirs to their parents.

The validity of the matching is essential to the credibility of the paper. For this reason, we perform several additional manual checks. First, we use thepeerage.com to check manually if individuals matched to their mother do not have siblings who were matched to their father. If this is the case, we re-match those to their fathers. Second, we calculate the distance between father's and children's surnames for individuals matched in iterations 2 to 4. To do so, we use the Levenshtein distance algorithm, which measures the minimum number of single-character edits required to change one surname into the other. We then use thepeerage.com to check manually all the matches with a Levenshtein distance above one.

Overall, we match 98.25 percent of the 26,499 entries in the dataset to their parents. Only 2.22 percent of them are matches to the mother.

Figure 4: James Hamilton, 1st Earl of Abercorn, Hollingsworth database.



Hollingsworth's Peerage Data — Ref No: 0 — New Record

Ref No: 0 — Child: S — Parent: 1B Pai — Rank: 1E — Title: Abercorn(S)

Surname: Hamilton — Comment:

First Names: James

Highest titles succeeded to.

Father Succ. 1 Heir 1 Cr. 8 — Mother Succ. 1 Heir 1 Cr. 1 — Self Succ. 1 Heir 8 Cr. 5

Illegitimate: Males 0, Females 0 — Died — Created: 10 July 160 — Violent Death — Notes: dvp

Sex/Death: 1 — Sole Heirship: X

	Day	Month	Year	Acc	After	Before	Comment
Birth	12	5	1575	y			
Death	23	0	1618	y			

N of Marriages 1 — No of this Marriage 1

Children: This Marriage, Live 9 — All Marriages, STILL — All Marriages, LIVE 9

	First Names	Surname	Comment	Child	Parent	Rank	Title	Address
Spouse	Marion		eld. dau.					
Widow/er of								
Spouse's Father	Thomas	Boyd				6B	Boyd	Kilmarnock
Spouse's Mother	Margaret or Mariar							
Mat. Gr'Father	Matthew	Campbell				Sir		London

Origin: Parent 8, Spouse 1 — Heir 3 — Notes:

	Marriage	Spouse's Birth	Spouse's Death	Divorce
Day	12	1	26	
Month	x	1	5	
Year	1599	1579	1632	
Acc.	6	7	y	
After				
Before				
Comment				

Children — Set No: 0 — Average accuracy of birth dates 6

Num	Name	Remarks	Da	Montl	Yea	Surviva	Accura	Ref Nc
1	Anne		19	9	1599	1	5	0
2	James	2E	22	x	1601	0	5	0
3	Claude	2B Strabane	21	2	1602	0	5	0
4	William		16	6	1603	0	5	0
5	George		9	x	1605	0	5	0
6	Margaret		28	4	1606	1	6	0
7	Lucy		11	x	1608	1	6	0
8	Isabel		20	6	1609	1	6	0
9	Archibald		24	2	1611	0	6	0
0					0			0

Record: 1 of 9 — No Filter — Search

B Additional figures and tables

Childlessness

	1650-99	1700-49	1750-99	1800-49	1850-99
Lévy and Henry (1960)[a]	9%	21%	35%	-	-
Ducs et pairs de France	(N=34)	(N=24)	(N=20)		
Pedlow (1982)[b]	5%	14%	9%	8%	8%
Nobility of Hesse-Kassel	(N=39)	(N=51)	(N=56)	(N=121)	(N=84)
This study:					
Peers' daughters[b]	40%	30%	32%	25%	18%
	(N=603)	(N=493)	(N=603)	(N=972)	(N=1,278)
Peers and peers' sons[b]	22%	26%	22%	20%	20%
	(N=492)	(N=493)	(N=627)	(N=1,057)	(N=1,391)

Notes: The sample are: *a*) women marrying before 20 years old whose marriage remained intact because neither spouse died before 45 years old; *b*) marriages that remained intact at least until the wife reached age 45.

Table 8: Comparison with other nobilities

Dep. Variable: Childlessness

	(1)	(2)	(3)	(4)	(5)
	IV triangular	IV classic			
	heirs	heirs	non-heirs	England and Ireland	Scotland
Settlement	-0.144***	-0.145***	0.026	-0.165***	-0.008
[i.e., father died after wedding]	(0.036)	(0.035)	(0.060)	(0.050)	(0.077)
Controls	YES	YES	YES	YES	YES
Family FE	YES	YES	YES	YES	YES
Birth year FE	YES	YES	YES	YES	YES
M. quarter-century FE	YES	YES	YES	YES	YES
Father-in-law status	-	-	YES	-	-
Observations	1,531	1,504	1,258	1,139	365
% correctly predicted	91.0	90.9	55.8	55.8	59.8
F-stat from first-stage	23.0	27.5	23.1	15.8	3.3

Notes: Column 1 presents the results from the benchmark IV triangular model described in Section 3.3. Columns 2 to 5 present the results from a classic IV model including all covariates in the first stage. The sample are all marriages in 1650–1882 where the husband is heir to a peerage (cols. 1 & 2), the husband is not a heir and the wife is a peers' daughter (col. 3), the husband is heir to an English or Irish peerage (col. 4), and the husband is heir to a Scottish peerage (col. 5). Controls are: number of siblings of the husband, wife's age at marriage, spouses' age at death, history of stillbirths in the husband's family, and wife's social status. First-stage results not reported; Standard errors clustered by family in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 9: IV with all covariates in first-stage (1650-1882)

C Proofs

This appendix proves Propositions 1 to 11.

C.1 Proof of Proposition 1

We solve for the optimal levels of consumption and bequests by backward induction. Generation 3 chooses the level of consumption that maximizes (9) subject to $x_3 = k_3$, where k_3 follows from the choices of generation 2.

Generation 2 chooses consumption, x_2, and bequests, k_3, to maximize (7) subject to (8), given the level of bequests chosen by generation 1, k_2. The optimal choices depend on whether generation 2 has children or not. If $n_2 = 0$, the optimal solutions are $x_2 = x_2^* := k_2$ and $k_3 = 0$. If $n_2 = 1$, the optimal solutions are

$$x_2 = x_2^{**} := \frac{k_2}{1+\beta\delta}, \quad \text{and} \quad k_3 = x_3^{**} := \frac{\beta\delta k_2}{1+\beta\delta}.$$

Generation 1 chooses consumption, x_1, and the bequests, k_2, to maximize (5) subject to (6). If $n_1 = 0$, the optimal solutions are $x_1 = K$ and $k_2 = 0$. If $n_2 = 0$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x_1^* := \frac{K}{1+\beta\delta}, \quad \text{and} \quad k_2 = k_2^* := \frac{\beta\delta K}{1+\beta\delta}.$$

If $n_2 = 1$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x_1^{**} := \frac{K}{1+\beta\delta+\beta\delta^2}, \text{ and } k_2 = k_2^{**} := K - \frac{K}{1+\beta\delta+\beta\delta^2}.$$

Replacing k_2^* in x_2^*, and k_2^{**} in x_2^{**} and x_3^{**}, Proposition 1 summarizes the optimal conditions detailed above.

C.2 Proof of Proposition 2

Generation 3 is always childless, $n_3 = 0$.

For generation 2, f is the difference between the indirect utility when $n_2 = 1$ and the indirect utility when $n_2 = 0$.

$$f(k_2, \beta, \delta) = \ln\left(\frac{k_2}{1+\beta\delta}\right) + \beta\delta \ln\left(\frac{\beta\delta k_2}{1+\beta\delta}\right) - \ln(k_2).$$

The partial derivative with respect to k_2 is $f_{k_2} = \dfrac{\beta\delta}{k_2} > 0$.

For generation 1, g is the difference between the indirect utility when $n_1 = 1,\ n_2 = 0$ and the indirect utility when $n_1 = 0$. Given the optimal solution on consumptions given in Proposition 1, x_1^* and x_2^*,

$$g(K, \beta, \delta) = \ln\left(\frac{K}{1+\beta\delta}\right) + \beta\delta \ln\left(\frac{\beta\delta K}{1+\beta\delta}\right) - \ln(K).$$

The partial derivative is $g_K = \dfrac{\beta\delta}{K} > 0$.

For generation 1, h is the difference between the indirect utility when $n_1 = 1,\ n_2 = 1$ and the indirect utility when $n_1 = 0$. Given the optimal solution on consumptions given in Proposition 1, x_1^{**}, x_2^{**} and x_3^{**},

$$h(K, \beta, \delta) = \ln\left(\frac{K}{1+\beta\delta+\beta\delta^2}\right) + \delta\beta \ln\left(\frac{1+\delta}{1+\beta\delta}\ \frac{\beta\delta K}{1+\beta\delta+\beta\delta^2}\right) + \beta\delta^2 \ln\left(\frac{\beta(1+\delta)}{1+\beta\delta}\ \frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right) - \ln(K).$$

The partial derivative is $h_K = \dfrac{\beta\delta(1+\delta)}{K} > 0$.

C.3 Proof of Proposition 3

From Proposition 2, the functions g and h, compare the indirect utilities of generation 1 when $n_1 = 1$ and when $n_1 = 0$ at the optimal levels of x_1, x_2, and x_3 given in Proposition 1. The function f, compares the indirect utilities of generation 2 when $n_2 = 1$ and when $n_2 = 0$ at the optimal level of k_2 given in Proposition 1. The sign of these functions gives the SPE.

C.4 Proof of Proposition 4

Let $\Gamma := \beta \cdot \delta$. The conditions for a high fertility SPE can be written as:

$$f(k_2^{**}, \beta, \delta) \geq 0 \iff \mathcal{C}_1(\beta) := \ln \frac{\beta\Gamma(1+\Gamma)K}{(1+\Gamma)(\beta+\beta\Gamma+\Gamma^2)} + \Gamma \ln \frac{\beta\Gamma^2(1+\Gamma)K}{(1+\Gamma)(\beta+\beta\Gamma+\Gamma^2)} - \ln \frac{\beta\Gamma(1+\Gamma)K}{\beta+\beta\Gamma+\Gamma^2} \geq 0, \quad (12)$$

$$h(K, \beta, \delta) > 0 \iff \mathcal{C}_2(\beta) := \ln \frac{\beta K}{\beta+\beta\Gamma+\Gamma^2} + \Gamma \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma K}{\beta+\beta\Gamma+\Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta+\beta\Gamma+\Gamma^2} - \ln K > 0 \quad (13)$$

and

$$v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^{*}, x_2^{*}, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$
$$\mathcal{C}_3(\beta) := \ln \frac{\beta K}{\beta+\beta\Gamma+\Gamma^2} + \Gamma \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma K}{\beta+\beta\Gamma+\Gamma^2} + \frac{\Gamma^2}{\beta} \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta+\beta\Gamma+\Gamma^2} - \ln \frac{K}{1+\Gamma} - \Gamma \ln \frac{\Gamma K}{1+\Gamma} > 0. \quad (14)$$

For a constant Γ, conditions (12)-(14) only depend on β. We then need to show that $\frac{\partial \mathcal{C}_1(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_2(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_3(\beta)}{\partial \beta} < 0$. Computing the derivatives, we have:

$$\frac{\partial \mathcal{C}_1(\beta)}{\partial \beta} = -\frac{\Gamma^2}{(\beta+\Gamma)(\beta+\beta\Gamma+\Gamma^2)} < 0,$$

and

$$\frac{\partial \mathcal{C}_2(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_3(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln \frac{\beta+\Gamma}{1+\Gamma} \frac{\Gamma^2 K}{\beta+\beta\Gamma+\Gamma^2} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln x_3^{**} < 0,$$

since n_2 would be nil otherwise.

C.5 Proof of Proposition 5

We solve for the optimal levels of consumption and bequests by backward induction. Generation 3 chooses the level of consumption that maximizes (9) subject to $x_3 = k_3$, where k_3 is given by the choices of generation 1.

Generation 2 chooses the level of consumption that maximizes (7) subject to $x_2 = k_2$, where k_3 is given by the choices of generation 1.

Generation 1 chooses consumption, x_1, and bequests, k_2 and k_3 to maximize (5) subject to (10). If $n_1 = 0$, the optimal solutions are $x_1 = K$ and $k_2 = k_3 = 0$. If $n_2 = 0$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x_1^* := \frac{K}{1+\beta\delta}, \quad k_2 = k_2^* := \frac{\beta\delta K}{1+\beta\delta}, \quad \text{and} \quad k_3 = k_3^* := 0.$$

If $n_2 = 1$ and $n_1 = 1$, the optimal solutions are

$$x_1 = x_{1c}^{**} := \frac{K}{1+\beta\delta+\beta\delta^2}, \quad k_2 = k_{2c}^{**} := \frac{\beta\delta K}{1+\beta\delta+\beta\delta^2},$$

$$\text{and} \quad k_3 = k_{3c}^{**} := \frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}.$$

Replacing k_2^* and k_3^* in x_2^*, k_{2c}^{**} in x_{2c}^{**}, and k_{3c}^{**} in x_{3c}^{**}, Proposition 5 summarizes the optimal conditions detailed above.

C.6 Proof of Proposition 6

Generation 3 is always childless, $n_3 = 0$.

For generation 2, $\mathcal{F}$ is the difference between the indirect utility when $n_2 = 1$ and the indirect utility when $n_2 = 0$.

$$\mathcal{F}(k_3, \beta, \delta) = \ln(k_2) + \beta\delta \ln(k_3) - \ln(k_2) = \beta\delta \ln(k_3).$$

The partial derivative is $\mathcal{F}_{k_3} = \beta\delta \frac{1}{k_3} > 0$.

For generation 1, g is the difference between the indirect utility when $n_1 = 1$, $n_2 = 0$ and the indirect utility when $n_1 = 0$. Note that this function is equivalent to the one defined in the model without commitment. Hence, Proof C.2 shows that $h_K > 0$.

For generation 1, $\mathcal{H}$ is the difference between the indirect utility when $n_1 = 1$, $n_2 =$

1 and the indirect utility when $n_1 = 0$.

$$\mathcal{H}(K,\beta,\delta) = \ln\left(\frac{K}{1+\beta\delta+\beta\delta^2}\right) + \beta\delta\ln\left(\frac{\beta\delta K}{1+\beta\delta+\beta\delta^2}\right) + \beta\delta^2\ln\left(\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right) - \ln(K).$$

The partial derivative is $\mathcal{H}_K = \dfrac{\beta\delta(1+\delta)}{K} > 0$.

C.7 Proof of Proposition 7

From Proposition 6, the functions g and $\mathcal{H}$ compare the indirect utilities of generation 1 when $n_1 = 1$ and when $n_1 = 0$ at the optimal levels of x_1, x_2, and x_3 given in Proposition 5. The function $\mathcal{F}$, compares the indirect utilities of generation 2 when $n_2 = 1$ and when $n_2 = 0$ at the optimal level of k_2 given in Proposition 5. The sign of these functions gives the SPE.

C.8 Proof of Proposition 8

Let $\Gamma := \beta\cdot\delta$. The conditions for a high fertility SPE can be written as:

$$\mathcal{F}(k_{3c}^{**},\beta,\delta) \geq 0 \iff \mathcal{C}_{1c}(\beta) := \Gamma\ln\frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} \geq 0, \tag{15}$$

$$\mathcal{H}(K,\beta,\delta) > 0 \iff \mathcal{C}_{2c}(\beta) := \ln\frac{\beta K}{\beta(1+\Gamma)+\Gamma^2} + \Gamma\ln\frac{\beta\Gamma K}{\beta(1+\Gamma)+\Gamma^2} + \frac{\Gamma^2}{\beta}\ln\frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} - \ln K > 0 \tag{16}$$

and

$$v_1(x_{1c}^{**},x_{2c}^{**},x_{3c}^{**},n_1{=}1,n_2{=}1) > v_1(x_1^*,x_2^*,x_3{=}0,n_1{=}1,n_2{=}0) \iff$$
$$\mathcal{C}_{3c}(\beta) := \ln\frac{\beta K}{\beta(1+\Gamma)+\Gamma^2} + \Gamma\ln\frac{\beta\Gamma K}{\beta(1+\Gamma)+\Gamma^2} + \frac{\Gamma^2}{\beta}\ln\frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} - \ln\frac{K}{1+\Gamma} - \Gamma\ln\frac{\Gamma K}{1+\Gamma} > 0. \tag{17}$$

Keeping Γ constant, conditions (15)-(17) only depend on β. We then need to show that $\frac{\partial \mathcal{C}_{1c}(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_{2c}(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_{3c}(\beta)}{\partial \beta} < 0$. Computing the derivatives, we then have:

$$\frac{\partial \mathcal{C}_{1c}(\beta)}{\partial \beta} = -\frac{\Gamma(1+\Gamma)}{\beta + \Gamma(\Gamma+\beta)} < 0,$$

and

$$\frac{\partial \mathcal{C}_{2c}(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_{3c}(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln \frac{\Gamma^2 K}{\beta(1+\Gamma)+\Gamma^2} = -\left(\frac{\Gamma}{\beta}\right)^2 \ln x_3^{**} < 0.$$

C.9 Proof of Proposition 9

We need to show that $\mathcal{F}(k_{3c}^{**}, \beta, \delta) - f(k_2^{**}, \beta, \delta) > 0$, $\mathcal{H}(K, \beta, \delta) - h(K, \beta, \delta) > 0$, and $v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) - v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) > 0$ for all β and δ in $[0, 1]$. First, note that:

$$\mathcal{F}(k_{3c}^{**}, \beta, \delta) - f(k_2^{**}, \beta, \delta) = \beta \ln \frac{1+\beta\delta}{\beta(1+\delta)} - \ln \frac{1}{1+\beta\delta}$$

where $\ln \frac{1+\beta\delta}{\beta(1+\delta)} \geq 0$ and $\ln \frac{1}{1+\beta\delta} \leq 0$. Hence, $\mathcal{F}(k_{3c}^{**}, \beta, \delta) - f(k_2^{**}, \beta, \delta) \geq 0$.

Second, note that:

$$\mathcal{H}(K, \beta, \delta) - h(K, \beta, \delta) = \beta\delta\mathcal{A}(\beta, \delta)$$

where $\mathcal{A}(\beta, \delta) := \ln \frac{1+\beta\delta}{1+\delta} + \delta \ln \frac{1+\beta\delta}{\beta(1+\delta)}$. The partial derivatives of $\mathcal{A}(\beta, \delta)$ are:

$$\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \beta} = -\frac{\delta(1-\beta)}{\beta(1+\beta\delta)} \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} = -\frac{1-\beta}{1+\beta\delta} + \ln \frac{1+\beta\delta}{\beta(1+\delta)} \geq 0.$$

To see why the second derivative is (weakly) positive, note that

$$\frac{\partial^2 \mathcal{A}(\beta, \delta)}{\partial \delta \partial \beta} = -\frac{1-\beta}{\beta(1+\beta\delta)^2} \leq 0 \quad \text{and} \quad \frac{\partial^2 \mathcal{A}(\beta, \delta)}{\partial \delta^2} = -\frac{1-\beta}{(1+\delta)(1+\beta\delta)^2} \leq 0.$$

This implies that the $\underset{\beta,\delta\in[0,1]}{\operatorname{argmin}} \frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} = \frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta}(1, 1) = 0$. In addition, $\lim \frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta}\Big|_{\beta \to 0} = +\infty$, which implies that $\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} \geq 0$.

Given that $\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \beta} \leq 0$ and $\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} \geq 0$, the $\underset{\beta,\delta\in[0,1]}{\operatorname{argmin}} \mathcal{A}(\beta, \delta) = \mathcal{A}(1, 0) = 0$.

In addition, $\lim \mathcal{A}(\beta,\delta)_{\beta\to 0} = +\infty$, which implies that $\mathcal{A}(\beta,\delta) \geq 0$ for all β and δ in $[0,1]$.

Third, note that:

$$\mathcal{H}(K,\beta,\delta) - h(K,\beta,\delta) = v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1{=}1, n_2{=}1) - v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1).$$

This concludes the proof.

C.10 Proof of Proposition 10

For any fixed value of $\Gamma := \beta \cdot \delta$, we need to show that:

$$\frac{\partial(\mathcal{C}_{1c} - \mathcal{C}_1)}{\partial\beta} < 0, \quad \frac{\partial(\mathcal{C}_{2c} - \mathcal{C}_2)}{\partial\beta} < 0, \quad \text{and} \quad \frac{\partial(\mathcal{C}_{3c} - \mathcal{C}_3)}{\partial\beta} < 0,$$

where $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ are the conditions for a high-equilibrium SPE in the model without commitment defined in (12)-(14) and $\mathcal{C}_{1c}$, $\mathcal{C}_{2c}$ and $\mathcal{C}_{3c}$ are the conditions for a high-equilibrium SPE in the model with commitment defined in (15)-(17). Computing the three derivatives we have,

$$\frac{\partial(\mathcal{C}_{1c} - \mathcal{C}_1)}{\partial\beta} = -\frac{\Gamma}{\Gamma+\beta} < 0 \quad \text{and} \quad \frac{\partial(\mathcal{C}_{2c} - \mathcal{C}_2)}{\partial\beta} = \frac{\partial(\mathcal{C}_{3c} - \mathcal{C}_3)}{\partial\beta} = \left(\frac{\Gamma}{\beta}\right)^2 \ln\frac{\beta+\Gamma}{1+\Gamma} < 0.$$

C.11 Proof of Proposition 11

Generation 1 is better off in the model with commitment as the condition

$$v_1(x_{1c}^{**}, x_{2c}^{**}, x_{3c}^{**}, n_1 = 1, n_2 = 1) > v_1(x_1^{*}, x_2^{*}, x_3 = 0, n_1 = 1, n_2 = 0) \tag{18}$$

defines the region characterized in Proposition 9. Note that condition (18) can be rewritten as:

$$\frac{1+\beta\delta}{\delta}\ln\left(\frac{1+\beta\delta}{1+\beta\delta+\beta\delta^2}\right) + \beta\delta\ln\left(\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right) > 0. \tag{19}$$

Generation 2 is better off in the model with commitment in the region characterized in Proposition 9 if and only if

$$v_2(x_{2c}^{**}, x_{3c}^{**}, n_2 = 1) > v_2(x_2^{*}, x_3 = 0, n_2 = 0)$$

which holds if and only if

$$\ln\left(\frac{1+\beta\delta}{1+\beta\delta+\beta\delta^2}\right)+\beta\delta\ln\left(\frac{\beta\delta^2 K}{1+\beta\delta+\beta\delta^2}\right)>0. \tag{20}$$

Inequality (19) implies that inequality (20) is satisfied.

D Extension with more than three generations

In this appendix, we relax the assumption that the dynasty dies after generation 3. We assume the opposite scenario in which the dynasty does not die at all if generation 2 chooses to have positive fertility. The utility of generation 1 provided in Equation (5) can therefore be rewritten as follows

$$v_1(x_1, x_2, k_3, n_1, n_2) = \ln(x_1) + n_1 \cdot \left[\beta\delta \ \ln(x_2) + n_2 \cdot \beta\delta^2 \ V(k_3)\right], \tag{21}$$

where,

$$V(k_3) := \ln(f(k_3)) + \delta \ln(f(k_3)) + \dots + \delta^{n-3} \ln(f(k_3)) + \dots$$

and $f(k_3)$ determines the consumption of generations $i = \{3, 4, 5, \dots\}$ as a function of the bequest k_3.

We assume that $f(k_3) = \alpha k_3$. That is, we assume that, in the long run, the (residual) family wealth k_3 generates a return of $(1 + \alpha)k_3$. Every future generation $i = \{3, 4, 5, \dots\}$ then consumes αk_3 and passes down k_3 as a bequest for the next generation. Under this assumption, $V(k_3)$ can be rewritten as:

$$V(k_3) = \left(\frac{1}{1-\delta}\right) \ln(\alpha k_3) \ .$$

The utility functions of generations 1 and 2 can be written as, respectively:

$$v_1(x_1, x_2, x_3, n_1, n_2) = \ln(x_1) + n_1 \cdot \left[\beta\delta \ \ln(x_2) + n_2 \cdot \beta\delta^2 \ \left(\frac{1}{1-\delta}\right) \ln(x_3)\right], \tag{22}$$

and

$$v_2(x_1, x_2, x_3, n_1, n_2) = \ln(x_2) + n_2 \cdot \beta\delta \ \left(\frac{1}{1-\delta}\right) \ln(x_3). \tag{23}$$

For the model without commitment where generation 1 chooses the bequests for generation 2 and generation 2 chooses the bequests for generation 3 (and all the remaining generations), we have the following optimal levels of consumption summarized in Proposition I.

Proposition I (Consumption and bequests without commitment) *Suppose that each generation decides over the bequests for the next generation. In any SPE:*

(a) *If $n_1 = 0$, generation 1 consumes all the dynasty wealth, $x_1 = K$.*

(b) *If $n_1 = 1$ and $n_2 = 0$, generation 1 consumes x_1^*, generation 2 consumes x_2^*, and* *generation 1 gives a bequest* k_2^*, *where* x_1^*, x_2^*, k_2^* *are defined in Proposition 1.*

(c) *If* $n_1 = 1$ *and* $n_2 = 1$, *generation 1 consumes* $x_1^{'} := \dfrac{(1-\delta)K}{1-(1-\beta)\delta}$, *generation 2 consumes* $x_2' := \dfrac{\beta\delta(1-\delta)K}{(1-(1-\beta)\delta)^2}$, *all future generations consume* $x_3' := \dfrac{\alpha(\beta\delta)^2 K}{(1-(1-\beta)\delta)^2}$, *and generations 1 and 2 give a bequest* $k_2' {:=} K - x_1'$ *and* $k_3' {:=} \dfrac{x_3'}{\alpha}$ *respectively.*

Proof: The proof follows that in Appendix C.1. ■

Propositions II and III generalize Propositions 2 and 3 as follows:

Proposition II (Fertility without commitment) *Suppose each generation decides over the bequests for the next generation. In any SPE:*

(b) *Generation 2 has children,* $n_2 = 1$, *if and only if:*

$$f(k_2, \beta, \delta) := v_2\left(x_2{=}\frac{(1-\delta)k_2}{1-\delta+\beta\delta}, x_3{=}\frac{\alpha\beta\delta k_2}{1-\delta+\beta\delta}, n_2{=}1\right) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0) > 0,$$

where $f'_{k_2} > 0$.

(c) *Generation 1 has children,* $n_1 = 1$, *if and only if:*

$$g(K, \beta, \delta) := v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) - v_1(x_1{=}K, x_2{=}0, x_3{=}0, n_1{=}0, n_2{=}0) > 0 \quad \textit{when } f(k_2^*, \beta, \delta) < 0,$$

or

$$h(K, \beta, \delta) := v_1(x_1', x_2', x_3', n_1{=}1, n_2{=}1) - v_1(x_1{=}K, x_2{=}0, x_3{=}0, n_1{=}0, n_2{=}0) > 0 \quad \textit{when } f(k_2', \beta, \delta) > 0$$

where $g_K, h_K > 0$.

Proof: The proof follows that in Appendix C.2. ■

Proposition III (SPE without commitment) *Suppose each generation decides over the bequests for the next generation. Then,*

(i) *A high-fertility strategy* $\{k_2', k_3', x_1', x_2', x_3', n_1{=}1, n_2{=}1, n_3{=}1\}$ *is the SPE if:*

(a) $f(k_2', \beta, \delta) \geq 0$; $h(K, \beta, \delta) > 0$; *and* *(b)* $v_1(x_1', x_2', x_3', n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $f(k_2^*, \beta, \delta) < 0$ *and* $f(k_2', \beta, \delta) > 0$.

(ii) *A low-fertility strategy* $\{k_2^*, k_3{=}0, x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0, n_3{=}0\}$ *is the SPE if:*

(a) $f(k_2^*, \beta, \delta) < 0$*;* $g(K, \beta, \delta) > 0$*; and*

(b) $v_1(x_1', x_2', x_3', n_1{=}1, n_2{=}1) \leq v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $f(k_2^*, \beta, \delta) < 0$ *and* $f(k_2', \beta, \delta) > 0$.

(iii) *A no-fertility strategy* $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}n_3{=}0\}$ *is the SPE if* $g(K, \beta, \delta) \leq 0$ *and* $h(K, \beta, \delta) \leq 0$.

Proof: The proof follows that in Appendix C.3. ■

The last proposition we need to show in order to conclude that all the results for the model without commitment remain valid when the dynasty does not die after generation 3 is Proposition 4. We now have,

Proposition IV (Comparative statics without commitment) *Suppose each generation decides over the bequests for the next generation. The conditions for a high-fertility SPE* $\{k_2', k_3', x_1', x_2', x_3', n_1{=}1, n_2{=}1, n_3{=}0\}$ *are more likely to be satisfied for more hyperbolic discount functions.*

Proof: Let $\Gamma := \beta \cdot \delta$. The conditions for a high fertility SPE can be written as:

$$f(k_2', \beta, \delta) \geq 0 \iff \mathcal{C}_1(\beta) := \ln \frac{\Gamma K \left(1 - \frac{X}{\beta}\right)}{\left(1 - \frac{\Gamma}{\beta} + \Gamma\right)^2} + \frac{\Gamma}{1 - \frac{\Gamma}{\beta}} \ln \frac{\alpha K \Gamma^2}{\left(1 - \frac{\Gamma}{\beta} + \Gamma\right)^2} - \ln \frac{\Gamma K}{1 - \frac{\Gamma}{\beta} + \Gamma} \geq 0,$$

$$h(K, \beta, \delta) > 0 \iff \mathcal{C}_2(\beta) := \ln \frac{\left(1 - \frac{\Gamma}{\beta}\right) K}{1 + \Gamma - \frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma K \left(1 - \frac{\Gamma}{\beta}\right)}{\left(1 + \Gamma - \frac{\Gamma}{\beta}\right)^2} + \frac{\frac{\Gamma^2}{\beta} \ln \frac{\alpha \Gamma^2 K}{\left(1 + \Gamma - \frac{\Gamma}{\beta}\right)^2}}{1 - \frac{\Gamma}{\beta}} - \ln K > 0$$

and

$$v_1(x_1^{**}, x_2^{**}, x_3^{**}, n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$

$$\mathcal{C}_3(\beta) := \ln \frac{\left(1-\frac{\Gamma}{\beta}\right)K}{1+\Gamma-\frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma K\left(1-\frac{\Gamma}{\beta}\right)}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2} + \frac{\frac{\Gamma^2}{\beta}\ln\frac{\alpha\Gamma^2 K}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2}}{1-\frac{\Gamma}{\beta}} - \ln\frac{K}{1+\Gamma} - \ln\frac{\Gamma K}{1+\Gamma} > 0.$$

As in Appendix C.4, we then need to show that $\frac{\partial \mathcal{C}_1(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_2(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_3(\beta)}{\partial \beta} < 0$. Computing the derivatives, we have:

$$\frac{\partial \mathcal{C}_1(\beta)}{\partial \beta} = -\frac{\Gamma^2\left(\beta-\Gamma+(\beta+\beta\Gamma-\Gamma)\ln\frac{\alpha\Gamma^2K}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2}\right)}{(\Gamma-\beta)^2(\beta+\beta\Gamma-\Gamma)} < 0,$$

and

$$\frac{\partial \mathcal{C}_2(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_3(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\Gamma-\beta}\right)^2\left(\frac{\Gamma(\beta-\Gamma)(1-\beta)}{\beta(\beta+\Gamma\beta-\Gamma)} + \ln\frac{\alpha\Gamma^2K}{\left(1+\Gamma-\frac{\Gamma}{\beta}\right)^2}\right) < 0,$$

as $\ln \frac{\alpha\Gamma^2 K}{\left(1-\frac{\Gamma}{\beta}+\Gamma\right)^2} = \ln(x_3')$ must be positive in order to satisfy $f(k_2', \beta, \delta) \geq 0$. ■

We now turn to the model with commitment and start with the optimal decisions when the dynasty does not die out.

Proposition V (Consumption and bequests with commitment) *Suppose generation 1 decides over the bequests for the following two generations. In any SPE:*

(*i*) *If* $n_1 = 0$, *generation 1 consumes all the dynasty wealth,* $x_1 = K$.

(*ii*) *If* $n_1 = 1$ *and* $n_2 = 0$, *generations 1 and 2 consume* x_1^* *and* x_2^* *and generation 1 gives a bequest* $k_2 = x_2^*$ *as in the model without commitment.*

(*iii*) *If* $n_1 = 1$ *and* $n_2 = 1$, *generation 1 consumes* $x_{1c}' := \frac{(1-\delta)K}{1-(1-\beta)\delta}$, *generation 2 consumes* $x_{2c}' := \frac{\beta\delta(1-\delta)K}{1-(1-\beta)\delta}$, *generation 3 consumes* $x_{3c}' := \frac{\alpha\beta\delta^2 K}{1-(1-\beta)\delta}$, *and generation 1 chooses* $k_{2c}' := x_{2c}'$ *and* $k_{3c}' := \frac{x_{3c}'}{\alpha}$ *as bequests.*

Proof: The proof follows that in Appendix C.5. ■

The following propositions generalize Propositions 6 and 7.

Proposition VI (Fertility with commitment) *Suppose that generation 1 decides over the bequests for the following generations. In any SPE:*

(*ii*) *Generation 2 has children, $n_2 = 1$, if and only if:*

$$\mathcal{F}(k_3, \beta, \delta) := v_2(x_2{=}k_2, x_3{=}\alpha k_3, n_2{=}1) - v_2(x_2{=}k_2, x_3{=}0, n_2{=}0) > 0,$$

where $\mathcal{F}_{k_3} > 0$.

(*iii*) *Generation 1 has children, $n_1 = 1$, if and only if:*

$$g(K, \beta, \delta) := v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0) - v_1(x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0) > 0$$

or

$$\mathcal{H}(K, \beta, \delta) := v_1(x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1) - v_1(x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}0) > 0 \text{ when } \mathcal{F}(k'_{3c}, \beta, \delta) > 0,$$

where $g_K, \mathcal{H}_K > 0$.

Proof: The proof follows that in Appendix C.6. ■

Proposition VII (SPE with commitment) *Suppose that generation 1 decides over the bequests for the following two generations. Then,*

(*i*) *A high-fertility strategy $\{k'_{2c}, k'_{3c}, x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1, n_3{=}1\}$ is the SPE if:*

- (*a*) $\mathcal{F}(k'_{3c}, \beta, \delta) \geq 0$; $\mathcal{H}(K, \beta, \delta) > 0$; *and*
- (*b*) $v_1(x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1) > v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$,

(*ii*) *A low-fertility strategy $\{k_2^*, k_3{=}0, x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0, n_3{=}0\}$ is the SPE if:*

- (*a*) $g(K, \beta, \delta) > 0$, *and*
- (*b*) $v_1(x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1) \leq v_1(x_1^*, x_2^*, x_3{=}0, n_1{=}1, n_2{=}0)$ *when* $\mathcal{F}(k'_{3c}, \beta, \delta) > 0$,

(*iii*) *A no-fertility strategy $\{k_2{=}k_3{=}0, x_1{=}K, x_2{=}x_3{=}0, n_1{=}n_2{=}n_3{=}0\}$ is the SPE if $g(K, \beta, \delta) \leq 0$ and $\mathcal{H}(K, \beta, \delta) \leq 0$.*

Proof: The proof follows that in Appendix C.7. ■

Proposition VIII generalizes Proposition 8.

Proposition VIII (Comparative statics with commitment) *Suppose generation 1 decides over the bequests for the following generations. The conditions for a high-fertility SPE* $\{k'_{2c}, k'_{3c}, x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1, n_3{=}0\}$ *are more likely to be satisfied for more hyperbolic discount functions.*

Proof: Let $\Gamma := \beta \cdot \delta$. The conditions for a high fertility SPE can be written as:

$$\mathcal{F}(k'_{3c}, \beta, \delta) \geq 0 \iff \mathcal{C}_{1c}(\beta) := \Gamma \ln \frac{\alpha \frac{\Gamma^2}{\beta} K}{1 + \Gamma - \frac{\Gamma}{\beta}} \geq 0,$$

$$\mathcal{H}(K, \beta, \delta) > 0 \iff \mathcal{C}_{2c}(\beta) := \ln \frac{\left(1 - \frac{\Gamma}{\beta}\right) K}{1 + \Gamma - \frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma \left(1 - \frac{\Gamma}{\beta}\right) K}{1 + \Gamma - \frac{\Gamma}{\beta}} + \frac{\Gamma^2}{\beta} \ln \frac{\alpha \frac{\Gamma^2}{\beta} K}{1 + \Gamma - \frac{\Gamma}{\beta}} - \ln K > 0$$

and

$$v_1(x^{**}_{1c}, x^{**}_{2c}, x^{**}_{3c}, n_1{=}1, n_2{=}1) > v_1(x^*_1, x^*_2, x_3{=}0, n_1{=}1, n_2{=}0) \iff$$

$$\mathcal{C}_{3c}(\beta) := \ln \frac{\left(1 - \frac{\Gamma}{\beta}\right) K}{1 + \Gamma - \frac{\Gamma}{\beta}} + \Gamma \ln \frac{\Gamma \left(1 - \frac{\Gamma}{\beta}\right) K}{1 + \Gamma - \frac{\Gamma}{\beta}} + \frac{\Gamma^2}{\beta} \ln \frac{\alpha \frac{\Gamma^2}{\beta} K}{1 + \Gamma - \frac{\Gamma}{\beta}} - \ln \frac{K}{1 + \Gamma} - \Gamma \ln \frac{\Gamma K}{1 + \Gamma} > 0.$$

We then need to show that $\frac{\partial \mathcal{C}_{1c}(\beta)}{\partial \beta} < 0$, $\frac{\partial \mathcal{C}_{2c}(\beta)}{\partial \beta} < 0$, and $\frac{\partial \mathcal{C}_{3c}(\beta)}{\partial \beta} < 0$. Computing the derivatives, we then have:

$$\frac{\partial \mathcal{C}_{1c}(\beta)}{\partial \beta} = -\frac{\Gamma}{(\Gamma - \beta)^2} \left(\frac{(1 + \Gamma)(\beta - \Gamma)\beta}{\beta + \beta\Gamma - \Gamma} + \Gamma \ln \frac{\alpha \Gamma^2 K}{\beta + \beta\Gamma - \Gamma} \right) < 0,$$

and

$$\frac{\partial \mathcal{C}_{2c}(\beta)}{\partial \beta} = \frac{\partial \mathcal{C}_{3c}(\beta)}{\partial \beta} = -\left(\frac{\Gamma}{\Gamma - \beta} \right)^2 \ln \frac{\alpha \Gamma^2 K}{\beta + \beta\Gamma - \Gamma} < 0.$$

■

The remaining propositions compare the two models. Propositions 9, 10 and 11 write exactly the same as in the benchmark model. The proofs when the dynasty does not to die after generation 3 are given below.

Proof of Proposition 9:

We need to show that $\mathcal{F}(k'_{3c}, \beta, \delta) - f(k'_2, \beta, \delta) > 0$, $\mathcal{H}(K, \beta, \delta) - h(K, \beta, \delta) > 0$, and $v_1(x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1) - v_1(x'_1, x'_2, x'_3, n_1{=}1, n_2{=}1) > 0$ for all β and δ in $[0, 1]$. First, note that:

$$\mathcal{F}(k'_{3c}, \beta, \delta) - f(k'_2, \beta, \delta) = \ln \frac{1-(1-\beta)\delta}{1-\delta} - \frac{\beta\delta}{1-\delta} \ln \frac{\beta}{1-(1-\beta)\delta}$$

where $\ln \frac{1-(1-\beta)\delta}{1-\delta} \geq 0$ and $\ln \frac{\beta}{1-(1-\beta)\delta} \leq 0$. Hence, $\mathcal{F}(k'_{3c}, \beta, \delta) - f(k'_2, \beta, \delta) \geq 0$.

Second, note that:

$$\mathcal{H}(K, \beta, \delta) - h(K, \beta, \delta) = \beta\delta\mathcal{A}'(\beta, \delta)$$

where $\mathcal{A}'(\beta, \delta) := \ln(1-(1-\beta)\delta) - \frac{\delta}{1-\delta} \ln \frac{\beta}{1-(1-\beta)\delta}$. The partial derivatives of $\mathcal{A}'(\beta, \delta)$ are:

$$\frac{\partial \mathcal{A}'(\beta, \delta)}{\partial \beta} = -\frac{(1-\beta)\delta}{\beta(1-(1-\beta)\delta)} \leq 0$$

and

$$\frac{\partial \mathcal{A}(\beta, \delta)}{\partial \delta} = \frac{1}{(1-\delta)^2} \left(\frac{\beta}{1-(1-\beta)\delta} - \ln\left(\frac{\beta}{1-(1-\beta)\delta} \right) - 1 \right) \geq 0.$$

To see why the second derivative is (weakly) positive, note that $a - \ln a - 1 \geq 0$, $\forall a > 0$.

Note that $\mathcal{A}'(1, \delta) = 0$ and $\lim \mathcal{A}'(\beta, \delta)_{\delta \to 0} = 0$. This implies that $\mathcal{A}(\beta, \delta) \geq 0$ for all β and δ in $[0, 1]$.

Third, note that:

$$\mathcal{H}(K, \beta, \delta) - h(K, \beta, \delta) = v_1(x'_{1c}, x'_{2c}, x'_{3c}, n_1{=}1, n_2{=}1) - v_1(x'_1, x'_2, x'_3, n_1{=}1, n_2{=}1).$$

This concludes the proof. ■

Proof of Proposition 10:

The proof follows Appendix C.10. For any fixed value of $\Gamma := \beta \cdot \delta$, we need to show that:

$$\frac{\partial(\mathcal{C}_{1c} - \mathcal{C}_1)}{\partial \beta} < 0, \quad \frac{\partial(\mathcal{C}_{2c} - \mathcal{C}_2)}{\partial \beta} < 0, \quad \text{and} \quad \frac{\partial(\mathcal{C}_{3c} - \mathcal{C}_3)}{\partial \beta} < 0,$$

Computing the first derivative we have,

$$\frac{\partial(\mathcal{C}_{1c}-\mathcal{C}_1)}{\partial\beta}=\frac{\Gamma}{(\Gamma-\beta)^2}\left(\Gamma-\beta+\Gamma\ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}\right)<0$$

since $\Gamma-\beta<0$ and $\beta^2<\beta+\beta\Gamma-\Gamma$. Computing the second and third derivatives we have,

$$\frac{\partial(\mathcal{C}_{2c}-\mathcal{C}_2)}{\partial\beta}=\frac{\partial(\mathcal{C}_{3c}-\mathcal{C}_3)}{\partial\beta}=\left(\frac{\Gamma}{\Gamma-\beta}\right)^2\left(\frac{\Gamma(\beta-\Gamma)(1-\beta)}{\beta(\beta+\beta\Gamma-\Gamma)}+\ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}\right).$$

To analyze the sign of these derivatives, we first define

$$\mathcal{D}(\Gamma):=\Gamma(\beta-\Gamma)(1-\beta)+\beta(\beta+\beta\Gamma-\Gamma)\ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}$$

where we can check that

$$\frac{\partial\mathcal{D}(\Gamma)}{\partial\Gamma}=-(1-\beta)\left(2(\Gamma-\beta)+\beta\ln\frac{\beta^2}{\beta+\beta\Gamma-\Gamma}\right)>0$$

and that $\mathcal{D}(0)<0$ and $\mathcal{D}(1)=0$ for any $\beta\in(0,1)$. Hence, $\frac{\partial(\mathcal{C}_{2c}-\mathcal{C}_2)}{\partial\beta}=\frac{\partial(\mathcal{C}_{3c}-\mathcal{C}_3)}{\partial\beta}<0$. ■

Last, we show that welfare is also higher in the model with commitment than in the model without commitment (Proposition 11).

Proof of Proposition 11: Generation 1 is better off in the model with commitment as the condition

$$v_1(x'_{1c},x'_{2c},x'_{3c},n_1=1,n_2=1)>v_1(x_1^*,x_2^*,x_3=0,n_1=1,n_2=0)\qquad(24)$$

defines the region characterized in Proposition 9. Note that condition (24) can be rewritten as:

$$\frac{1+\beta\delta}{\delta}\ln\left(\frac{(1-\delta)(1+\beta\delta)}{1-(1-\beta)\delta}\right)+\frac{\beta\delta}{1-\delta}\ln\left(\frac{\alpha\beta\delta^2K}{1-(1-\beta)\delta}\right)>0.\qquad(25)$$

Generation 2 is better off in the model with commitment in the region characterized in Proposition 9 if and only if

$$v_2(x'_{2c},x'_{3c},n_2=1)>v_2(x_2^*,x_3=0,n_2=0)$$

which holds if and only if

$$\ln\left(\frac{(1-\delta)(1+\beta\delta)}{1-(1-\beta)\delta}\right)+\beta\delta\ln\left(\frac{\alpha\beta\delta^2K}{1-(1-\beta)\delta}\right)>0. \tag{26}$$

Inequality (25) implies that inequality (26) is satisfied. ■