Childbearing Postponement, its Option Value, and the Biological Clock

David de la Croix[1] and Aude Pommeret[2]

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Abstract

Having children is like investing in a risky project. Postponing birth is like delaying an irreversible investment. It has an option value, which depends on its costs and benefits, and in particular on the additional risks motherhood brings. We develop a parsimonious theory of childbearing postponement along these lines. We derive its implications for asset accumulation, income, optimal age at first birth, and childlessness. The structural parameters are estimated by matching the predictions of the model to data from the National Longitudinal Survey of Youth NLSY79. The uncertainty surrounding income growth is shown to increase with childbearing, and this increase is stronger for more educated people. This effect alone can explain why the age at first birth and the childlessness rate both increase with education. We use the model to simulate two hypothetical policies. Providing free medically assisted reproduction technology does not affect the age at first birth much, but lowers the childlessness rate. Insuring mothers against income risk is powerful in lowering the age at first birth.

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[1]IRES, Université catholique de Louvain and CEPR. Email: `david.delacroix@uclouvain.be`.
[2]SEE, City University of Hong Kong and IREGE Université Savoie Mont Blanc.
Email: `aude.pommeret@cityu.edu.hk`.
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1 Introduction

Having a child increases risk. This is especially true as far as future income is concerned. Uncertain career costs include the atrophy of skills due to random interruptions (Adda, Dustmann, and Stevens 2017), a lower probability of getting promoted from temporary to permanent jobs (Guner, Kaya, and Sanchez Marcos 2017), more frequent occupation and workplace changes (Lundborg, Plug, and Rasmussen 2016), lost earnings opportunities with possibly lower wages, and a possibility of discrimination (Correll, Benard, and Paik 2007). In addition, parents also endure an increase in sickness absence (Angelov, Johansson, and Lindahl 2013). This pattern is likely to be reinforced when children have special needs (such as visual or hearing impairment, or mental retardation).

Beyond the issues of income and career, there is increased uncertainty affecting spending and utility flows. Many examples can be found in the literature: childrearing reduces women's social network size and alters the composition of men's networks (Munch, McPherson, and Smith-Lovin 1997); childrearing may have long-term health consequences such as urinary incontinence, weight gain, etc; and having a baby causes a substantial decline in the average couple's relationship (Doss et al. 2009). The most extreme case of risk incurred when being a mother is of course that of maternal mortality. The consequences of this risk for fertility have been studied in detail: exploiting variations in mortality risks across US states and cohorts, Albanesi and Olivetti (2014) show that the growth in fertility was highest for US states and cohorts of women that experienced the greatest reduction in maternal mortality. Albanesi and Olivetti (2016) show that improvements in maternal health reducing maternal mortality and morbidity are important to explain the joint evolution of married women's participation and fertility in the United States during the twentieth century.

Although the literature is full of examples stressing this increase in uncertainty following the birth of a first child, it does not treat it as such (except for the maternal mortality risk). It indeed focuses on first-order moments – such as the effect of having a child on the mean wage, the employment rate, etc – without acknowledging the risk component. Miller (2011) finds that delaying motherhood leads to a substantial increase in labor market earnings, of 9% per year of delay. This benefit goes through an increase in wages of 3% and an increase in work hours of 6%. Herr (2016) looks at the specific effect of first birth on wages. For each woman, she measures the time in her labor market career when children are first present. For women who entered the labor market before having children, she finds a clear monotonic relationship between delayed first birth and higher long-run wages. Budig and England (2001) look at the effect of having children on wages and employment. They find a wage penalty for motherhood of approximately 7 % per child. One-third of the penalty is explained by years of past job experience and seniority, because motherhood interrupts women's employment, leading to breaks, more part-time work, and fewer years of experience and seniority. The authors guess that the remaining two-thirds of the motherhood penalty may arise from the impact of motherhood on productivity and/or from employer discrimination. Note that all these studies are based on the National Longitudinal Survey of Youth (NLSY79) which is the data set we use in our quantitative analysis as well.

In this paper, we develop a theory in which motherhood increases risk. We model the risky nature of procreation explicitly, and stress that it is of particular importance for the optimal age at childbearing. We focus on how to model increased risk, how to measure it in the data, and whether it matters for household choices. The main idea we develop is that if having a child is irreversible and affects expected future earnings through risk, waiting (postponing birth) has a value (option value). A robust result of option theory is that the riskier an investment project, the worthier it is to wait (Dixit and Pindyck 1994). In a different context, we also obtain that the option value of postponing birth increases with risk. Beyond income risks, the value of waiting interacts with fecundity (the biological clock) and the availability of assisted procreation techniques.

Our model has some of the same innovative characteristics as Adda, Dustmann, and Stevens (2017), namely skill atrophy, intertemporal budget constraint, and risk aversion. Apart from the risk aspect, their model is richer than ours (they also consider occupational choices and marital status) and needs to be solved numerically, using indirect inference and data on women born in Germany between 1955 and 1975. Our model is more parsimonious and allows for analytical resolution and therefore a clear grasp of the mechanisms. It can indeed be solved explicitly using stochastic optimal control and optimal control with regime switches (Boucekkine, Pommeret, and Prieur 2013). Our theory highlights how the timing of the first birth depends on financial uncertainty and on the risk of infertility. The model also allows to distinguish between three types of childlessness: voluntary, natural (primary sterility), and childlessness due to postponement.

We also conduct a quantitative analysis, identifying the structural parameters of the model using the National Longitudinal Survey of Youth (NLSY79). This survey follows the lives of a sample of American youth born between 1957-64 from Round 1 (1979 survey year) to Round 25 (2012 survey year). It started in 1979 with a sample of women aged 14 to 22, who were interviewed regularly from then on. Two-thirds of the sample was still observed at the end of the childbearing years, at which point 84 percent had children, which allows to study the effect and timing of childbearing on wages and employment. We show that mothers face a higher income risk than childless women. Although risk decreases with education, the risk differential between mothers and childless women increases with the education level, which partly explains why educated parents have children later.

Finally, we use the model to investigate the effect of two policies. First, introducing a hypothetical insurance against motherhood-related risks appears to be a very strong tool to reduce the age at first birth for the more educated. The empirical literature (see Gauthier (2007) for a survey, and d'Albis, Greulich, and Ponthière (2015)) suggests that well-designed public policy can affect the timing of fertility, including childcare provision and lump-sum financial incentives. In unequal societies, having a well-developed market for nannies and babysitters might play the same role (Hazan and Zoabi 2013). Second, we simulate the effect of free and highly effective medically assisted procreation, which amounts to make women three years younger. This policy delays the age at first birth by less than one year for the higher education categories, and reduces childlessness, but not more than the insurance policy. Our results on assisted procreation are in line with Sommer (2016) as she finds that the introduction of IVF technology (calibrated on 2012 IVF success rates) increases the number of births but is not sufficient to compensate for the effect of the increased earning risk observed on the period studied. On the whole, our results indicate that insurance against motherhood-related risks seems more effective than artificial procreation to advance births.

There exists a literature on the optimal timing of births. A first approach is deterministic and the dynamic structure is simple, with only a choice between early and late childbearing, as in Low (2013). In her model, women can trade one more year of job experience or training for having babies early in life (and getting married). The interest of the static structure is to allow to solve for equilibrium on the marriage market, and to study its properties analytically. Pestieau and Ponthière (2014, 2015) propose a dynamic model in discrete time in which parents can have children early or late (binary choice). Here again the simple dynamic structure allows to provide a general equilibrium analysis. An early dynamic model of fertility can be found in Heckman and Willis (1976). They focus on the proximate determinants of fertility. In their approach, it is costly not to have children (cost of contraception). The other costs are not modelled. Their model suggests that a woman's reproductive history depends on the sequence of contraception decisions a couple makes. The authors notice that "the optimal decision making that they have specified requires a couple to solve a stochastic dynamic programming problem at the beginning of each month from marriage to menopause." Later, Cigno and Ermisch (1989) focus on the interaction between physiological and financial considerations in a deterministic framework. The interactions between demographics and economics are studied by d'Albis, Augeraud Véron, and Schubert (2010) and de la Croix and Licandro (2013) in dynamic deterministic models in which women choose the time of birth. They show how the growth rate of the population is affected by this choice. Compared to all these approaches, we neglect general equilibrium effects and the marriage market aspect, but we model the time dimension more precisely, as the trade-off between fecundity and income depends crucially on age, and is not the same at 25, 35, or 40.

Even existing structural stochastic models do not explicitly make risk depend on motherhood. Francesconi (2002) and Sheran (2007) account for some uncertainty, but it takes the form of taste, technological shocks, and/or birth control shocks that are not affected by labor or fertility decisions. For instance, Francesconi (2002) estimates the structural parameters of a finite-horizon, discrete-choice model on a sample of married women from the National Longitudinal Survey (NLS) of Young Women (1968-1991), and show that a short interruption of full-time work is less harmful for the earnings profile than a part-time experience during childrearing. Using the same data set and the same type of model, Sheran (2007) shows that a childcare subsidy is likely to reduce women's education level, but increase their time spent working. It should be noted that even if these papers study the joint decision of female labor supply and fertility using dynamic life-cycle models, their objective is not to study childbearing decisions, but rather the consequences of children on labor-related choices in order to better predict the effect of public policies that are likely to affect both decisions. Sommer (2016) studies the decision to have children and accounts for earning risks, but again, childbirth does not affect risk: mothers and childless women face the same shocks and the same asset accumulation. Note however that due to motherhood, women may decide to spend less time at work, which in fact reduces their sensitivity to these shocks. In this case, having children provides insurance, which is in line with the "old age security" hypothesis (Nugent 1985) based on the idea that children are a security asset.[1] Sommer (2016) finds that having children is considered as a consumption commitment, and her model explains half of the decrease in the number of births between 1970 and 1990 when the US labor market risk was high.[2] In addition, she finds that fertility and earnings risks amplify each other as far as the number of births is concerned, even if the infertility risk leads women to have children earlier.

Demographers have also written extensively on childbearing postponement. When they aim at analyzing economic uncertainty, their preferred approach is to include unemployment rates as a forcing variable in their empirical studies (Hoem 2000, Meron and Widmer 2002, and Pailhé and Solaz 2012).

The paper is organized as follows. The theory is exposed in Section 2. The main analytical results are provided in Section 3. The quantitative part, including calibration simulation and policy, is in Section 4. Section 5 concludes.

[1] However, the empirical literature favors a negative effect of uncertainty on fertility, see Hofmann and Hohmeyer (2013) or Schneider (2015).

[2] This is consistent with the findings in Chabé-Ferret and Gobbi (2016) on post WWII data.

2 Theory

Time is continuous. The woman's life extends from time 0 to ∞. An infinite horizon is assumed for simplicity. Completed fertility can be either zero or one child. τ denotes the date when the woman starts trying to have children. Procreation succeeds at time τ with probability $\pi(\tau)$. If it fails, we assume for simplicity that there is no second chance (at age 30, 2/3 of conceptions occur within one year of the procreation attempt, see Léridon (2004)). With this assumption, all uncertainty surrounding fecundity is resolved at time τ. The probability π is decreasing in age τ and depends on medical technology.

We denote the natural sterility rate as: $\pi(0) = \bar{\pi}$. We also assume a menopause age T such that $\pi(T) = 0$. We assume that sterility is not affected by age for very young ages and for ages close to menopause: $\pi'(\tau) = 0$ for $\tau \leq 0$ or $\tau \geq T$.

The age at first birth is denoted θ. It is given by:

$$\theta = \begin{cases} \tau & \text{with proba.} \pi(\tau) \\ +\infty & \text{with proba.} 1 - \pi(\tau) \end{cases} \tag{1}$$

Women derive utility from consumption flow c and from having children. c is a composite good which includes both physical goods and leisure. Accounting for the child's consumption by adding a multiplicative term (larger than unity) to consumption after the child's birth would not alter the results significantly. The life-cycle utility when having a child at time τ is:

$$\int_0^\infty u(c_t)\, e^{-\rho t} dt + e^{-\rho\theta}\omega \tag{2}$$

where ω is the lump-sum utility of having children, and ρ is the psychological discount rate. $u(\cdot)$ is an increasing and concave function of consumption c_t. We focus on a woman's program as Lundborg, Plug, and Rasmussen (2016) have shown, using instrumental variable evidence from IVF treatments, that the effects of having a child on her partner's annual earnings are small, and much smaller than those estimated for women.

To get explicit analytical solutions, we assume instantaneous CRRA utility:[3]

$$u(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon}$$

[3]Note that a CRRA utility function features risk aversion as $u'' < 0$ and prudence as $u''' > 0$.

Parameter ε represents both the relative risk aversion and the inverse of the intertemporal elasticity of substitution. As in most of the literature on risk, we assume that $\varepsilon > 1$.

The woman starts her life with an initial wealth a_0, which is to be interpreted as including both physical wealth and experience capital. Asset dynamics follow Ito processes:

$$da_t = \begin{cases} (r_1 \ a_t - c_t)dt & \text{if } t \leq \theta \\ (r_2 \ a_t - c_t)dt + \sigma \ \ a_t \ dz_t & \text{otherwise} \end{cases} \tag{3}$$

which defines the budget constraint under which intertemporal utility (2) is maximized. We assume that wealth accumulation is deterministic until the child's birth. Income after birth is affected by dz_t, a Wiener process (Brownian motion) with $\mathbb{E}[dz_t] = 0$, $\text{var}[dz_t] = t$. The uncertainty parameter σ conveys the strength with which shocks affect wealth accumulation. The interest rates r_1 and r_2 denote the return on wealth for childless women and for mothers, respectively. They include both the return on human capital and the return on physical wealth.

Having a child has a level effect, through an overall lowering[4] of the mean return on assets $r_2 < r_1$, and a variance effect, through the inclusion of the Wiener process.[5] This is consistent with the results of Adda, Dustmann, and Stevens (2017) who show that the career cost is "a combination of occupational choice, lost earnings due to intermittency, lost investment into skills and atrophy of skills while out of work, and a reduction in work hours when in work." It is also in line with the returns to experience featured in the dynastic model of Gayle, Golan, and Soytas (2015), according to which working less after having a child reduces future earnings in a non-linear way since returns are not linear with the time spent working.

Each woman has an education level which may affect the deterministic part of the return on wealth. Education may also modify the excess volatility of the return on wealth of mothers compared to childless women. Hence, r_1, r_2 and σ are different across education levels.

The woman's problem is to choose a consumption savings plan a_t, c_t and a date τ at which she will start trying to have children. Her value function is given by

$$W(a_0) = \underset{c_t, a_t, \tau}{\arg\max} \ \mathbb{E}\left[\int_0^\infty u(c_t) \ e^{-\rho t} dt + e^{-\rho\theta}\omega\right]$$

[4]Unless specified otherwise: the case $r_2 \geq r_1$ will sometimes be considered later in the paper to get insights into the mechanisms of the model.

[5]Modeling a higher variance of shocks after some event (here birth) can be found in the macro-health literature. For example, in Capatina, Keane, and Maruyama (2017), the variance of income increases after a bad health shock which shifts health from a good to a bad state/regime.

where W expectations are taken with respect to the distribution of dz_t and θ, and the woman is subject to the budget constraint (3) and to the initial asset holding a_0.

We first provide the solution to the woman's standard problem when she decides from the beginning not to have children ($\tau = +\infty$). In this case, our problem is a standard textbook problem (see e.g. Barro and Sala-I-Martin (2001), p64-67):

$$(c_t, a_t) = \arg\max \mathbb{E}\left[\int_{\tau}^{\infty} \frac{c_t^{1-\varepsilon}}{1-\varepsilon}\, dt\right] \text{ subject to } da_t = (r_1\ a_t - c_t)dt, \text{ and } a_0 \text{ given,} \tag{4}$$

and subject to the usual transversality condition. The optimal dynamics for assets is:

$$a_t = a_0\ e^{\frac{r_1-\rho}{\varepsilon}\ t}. \tag{5}$$

and the initial consumption is given by $c_0 = p\ a_0$ where

$$p = \frac{\rho - (1-\varepsilon)r_1}{\varepsilon}. \tag{6}$$

p is the marginal propensity to consume out of initial wealth in the standard model. In this problem, the woman has forgone the option to procreate from the beginning.

Let us now consider the more general problem in which the woman has to decide when she will try to procreate. The problem has to be solved recursively:

- [**A**] Using stochastic optimal control (Turnovsky (2000)), we first consider the post-birth program, once the pregnancy attempt has proven successful. This delivers a utility $W_2(a_\tau)$ at a date τ with probability $\pi(\tau)$.
- [**B**] We also consider the case of a failed attempt to have children (this requires standard optimal control). This delivers a utility $W_1(a_\tau)$ at a date τ with probability $1 - \pi(\tau)$.
- [**C**] Finally, using optimal control with optimal regime switching (Boucekkine, Pommeret, and Prieur (2013)), we study the program starting from the beginning of her professional life, which includes the optimal choice of τ.

[A] The Post-Birth Program

The program is:

$$W_2(a_\tau) = \arg\max_{c_t, a_t} \mathbb{E}\left[\int_{\tau}^{\infty} u(c_t)\ e^{-\rho(t-\tau)}dt + \omega\right]$$

$$\begin{aligned} \text{subject to} \quad da_t &= (r_2\, a_t - c_t)dt + \sigma\, a_t\, dz_t \\ \tau, a_\tau & \quad \text{given.} \end{aligned}$$

The program is solved in Appendix A. Consumption follows

$$c_t = q a_t, \quad \forall t \geq \tau$$

with the propensity to consume out of wealth given by

$$q = \frac{\rho - (1-\varepsilon)\left(r_2 - \frac{\varepsilon}{2}\sigma^2\right)}{\varepsilon} \tag{7}$$

Here, we need to impose $\rho > (r_2 - \varepsilon\, \sigma^2/2)(1-\varepsilon)$ to guarantee positive consumption. Equation (7) shows that if we had considered a log utility function, the effect of uncertainty on the consumption/saving choice would have been ruled out. This is due to the fact that uncertainty, as it is modeled, affects the consumption/saving choice through the certainty-equivalent[6] asset growth $r_2 - \varepsilon\, \sigma^2/2$.

The value function is

$$W_2(a_\tau) = q^{-\varepsilon} \frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega. \tag{8}$$

Using the results in Dixit and Pindyck (1994), p72, the mean and variance of assets are:

$$\mathbb{E}\, a_t = a_\tau\, e^{(r_2-q)(t-\tau)}, \tag{9}$$

$$\text{Var}\, a_t = a_\tau^2\, e^{2(r_2-q)(t-\tau)} \left(e^{\sigma^2(t-\tau)} - 1\right), \tag{10}$$

and, since the percentage changes in a variable which follows a Brownian motion with drift are normally distributed, we have

$$d \ln a_T \sim \mathcal{N}\left(\left(r_2 - q - \frac{\sigma^2}{2}\right)(T-\tau), \sigma\, \sqrt{T-\tau}\right). \tag{11}$$

This distribution pertains to an individual forecasting her assets from time τ onwards, but also describes the distribution of wealth across individuals sharing the same parameters.

[6]We define the certainty-equivalent $\hat{X}(t{+}dt)$ of an uncertain variable $X(t{+}dt)$ as $\hat{X}(t{+}dt) = V^{-1}(E_t(V(X(t{+}dt))))$, where $V(X)$ accounts for the attitude with respect to risk. Here, $V(X) = \frac{X^{1-\varepsilon}}{1-\varepsilon}$.

[B] The Program in Case of Sterility at Age τ

The program is:

$$W_1(a_\tau) = \arg\max_{c_t, a_t} \mathbb{E}\left[\int_\tau^\infty u(c_t)\, e^{-\rho(t-\tau)}\, dt\right]$$

$$\begin{array}{rrcl} \text{subject to} & da_t & = & (r_1\, a_t - c_t)dt \\ & \tau, a_\tau & & \text{given.} \end{array}$$

By symmetry with the previous case, consumption follows

$$c_t = pa_t,$$

where the propensity to consume p is the same as in the benchmark program (4). We have $p > q$ as $\varepsilon > 1$.

The value function is

$$W_1(a_\tau) = p^{-\varepsilon} \frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}. \tag{12}$$

Assets are given by:

$$a_t = a_\tau\, e^{(r_1-p)(t-\tau)} = a_\tau\, e^{\frac{r_1-\rho}{\varepsilon}\,(t-\tau)}. \tag{13}$$

[C] The Full Program

The full maximization program can be written:

$$W(a_0) = \max_{\{c_t, \tau, a_t\}} \int_0^\tau u(c_t) e^{-\rho t} dt + \varphi(\tau, a_\tau)$$

$$\begin{array}{rcl} \text{where } \varphi(\tau, a_\tau) & = & e^{-\rho\tau}\left[\pi(\tau) W_2(a_\tau) + (1-\pi(\tau)) W_1(a_\tau)\right] \\ \text{with } W_2(a_\tau) & = & q^{-\varepsilon}\dfrac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega \;\text{ and }\; W_1(a_\tau) = p^{-\varepsilon}\dfrac{a_\tau^{1-\varepsilon}}{1-\varepsilon} \\ \text{subject to} & : & \dot{a}_t = r_1\, a_t - c_t \;\text{ and }\; a_0 \text{ given} \end{array}$$

There is no expectation operator in this program since all the uncertainty concerns what happens from date τ onwards, and expectations with respect to returns on future assets have already been computed in the previous step, while expectations with respect to birth are fully expressed using probability $\pi(\tau)$.[7]

To solve for the optimal choice, we follow the methodology proposed by Boucekkine, Pommeret, and Prieur (2013). We first define the following Hamiltonian:

$$H(c, a, \mu) = U(c)e^{-\rho t} + \mu\,(r_1\; a - c)$$

One can readily write the value-function $W(a_0)$ in terms of the Hamiltonian $H(\cdot)$:

$$W(a_0) = \int_0^{\tau} \left(H(c_t, a_t, \mu_t) - \mu_t\; \dot{a}_t\right) dt + \varphi(\tau, a_\tau)$$

We show in Appendix B that the first-order conditions are:

$$\begin{aligned} \frac{\partial H(c_t, a_t, \mu_t)}{\partial c_t} &= 0, & (14)\\ \frac{\partial H(c_t, a_t, \mu_t)}{\partial a_t} + \dot{\mu}_t &= 0, & (15)\\ H(c_\tau, a_\tau, \mu_\tau) + \frac{\partial \varphi(\tau, a_\tau)}{\partial \tau} &= 0, & (16)\\ \frac{\partial \varphi(\tau, a_\tau)}{\partial a_\tau} - \mu_\tau &= 0. & (17) \end{aligned}$$

The first two conditions (14) and (15) are the standard Pontryagin conditions. The last two conditions (16) and (17) may be interpreted as optimality conditions with respect to the switching time τ and the free state value a_τ. The third one, Equation (16), equalizes the marginal benefit of waiting to the marginal cost of waiting. The last one is a continuity condition: it implies that the shadow price of the state variable at the time of the switch, μ_τ, is equal to the expected marginal value of the state variable in τ (derived from the programs after the switch).

Conditions (14)-(16) are necessary but not sufficient for an interior maximum. Problems [A] and [B] both imply convex maximization programs. Problem [C] may admit a corner solution and the existence of an interior maximum must be checked numerically.

The time consistency of a policy $\{c_t, \tau, a_t\}$ decided at time 0 would imply its optimality at later stages t_0, t_1 (but still in the pre-birth part of the problem). Rewriting the maximization program as a decision made at time t_0 leading to policy $\{\hat{c}_{t-t_0}, \hat{\tau} - t_0, \hat{a}_{t-t_0}\}$, and one at time t_1 leading to $\{\bar{c}_{t-t_1}, \bar{\tau} - t_1, \bar{a}_{t-t_1}\}$, one can show using conditions (14)-(17) that $\hat{c}_t = \bar{c}_t$, $\hat{a}_t = \bar{a}_t$, and $\hat{\tau} = \bar{\tau}$. Initial conditions at t_0 and t_1 are supposed consistent here with the maximization

[7] Note that having an uncertain lump-sum utility of having children would not alter the nature of the problem, and ω would then simply be replaced by its expectation.

program at time 0. This result comes from the fact that the objective function is a discounted expected cumulative reward and discounting is exponential.

We show (see Appendix B) that conditions (14)-(17) allow to solve for the dynamics of the asset a_t and of consumption c_t as functions of time, and provide an implicit expression for the optimal procreation attempt date. In particular, Equation (17) allows to find assets and consumption at the time of the procreation attempt as a function of τ:

$$a_\tau = a_0\, e^{\frac{r_1-\rho}{\varepsilon}\tau} X(\tau), \tag{18}$$

$$c_\tau = a_0\; s(\tau)\; X(\tau) e^{\frac{r_1-\rho}{\varepsilon}\tau}, \tag{19}$$

with

$$X(\tau) = \frac{e^{p\tau}}{1 + s(\tau)\left[e^{p\tau} - 1\right]/p}, \tag{20}$$

$$s(\tau) = \left(\pi(\tau) q^{-\varepsilon} + (1 - \pi(\tau)) p^{-\varepsilon}\right)^{-1/\varepsilon}. \tag{21}$$

$s(\tau)$ is a CES function of the marginal propensity to consume of mothers and of voluntarily childless (or sterile) women. $X(\tau)$ is a factor stemming from the presence of the option to procreate. Indeed, if $\pi(\tau) = 0$ (sterility), $X(\tau) = 1$. We now turn to the interpretation of the results.

3 Interpretation and Results

3.1 Asset Accumulation

We will first look at asset accumulation. We consider four types of women: the voluntarily childless woman (type **V**), the sterile woman (type **S**), the candidate mother (type **C**), and the mother (type **M**).

The following proposition shows that women who intend to attempt to get pregnant accumulate more assets to smooth consumption in the face of the drop in the certainty-equivalent asset growth ($r_2 - \frac{\varepsilon}{2}\sigma^2 < r_1$).

Proposition 1 *Consider $s(\tau)$, the marginal propensity to consume the asset of a candidate mother (type* **C***).*

- *The higher the success rate $\pi(\tau)$, the lower $s(\tau)$.*
- *If success is certain ($\pi(\tau) = 1$), $s(\tau)$ is the same as that of type* **M** *women.*

◇ *If failure is certain ($\pi(\tau) = 0$), $s(\tau)$ is the same as that of type* **V** *women ($\tau = +\infty$).*

Proof: From Equation (21), $\frac{\partial s(\tau)}{\partial \pi(\tau)} < 0 \Leftrightarrow \varepsilon > 1$ and $s(\pi = 1) = q$, $s(\pi = 0) = p$. ■

These results are in line with Blundell, Pistaferri, and Saporta-Eksten (2017). Using a life-cycle approach with exogenous fertility decisions, they derive structural marginal rate of substitution relations between leisure time of the two spouses, and estimate a subset of the structural parameters of the model.[8] They argue that in the pre-children period, the household is "[...]saving in anticipation of the decline in family earnings induced by the wife reallocating time from market to childcare when children arrive".

It is also worthwhile to remark that precautionary savings decrease with the importance of the risk on the procreation side $\pi(\tau)$.

We can now compare the assets of a woman trying to procreate a_τ to those of type **V** women, given by Equation (5). a_τ is increased by the option to procreate as future and current consumption are gross complements. Women expecting to have a child accumulate more assets in order to face a decrease in the certainty-equivalent asset growth. This is similar to a "precautionary saving" effect except precautionary saving is usually defined as an increase in asset accumulation in the face of uncertainty affecting the next period (see Kimball (1990)) and the following ones. Here, uncertainty starts affecting returns $\tau - t$ periods later with, in addition, the date τ decided by the agent herself.

Corollary 1 *Before the procreation attempt, the asset growth rate of type* **S** *and* **M** *women is the same. The asset growth rate of type* **V** *women is smaller.*

Proof: The first part of the proposition is trivial as, before trying to procreate, **S** and **M** women are identical. From Appendix B, the dynamics of their assets is given by $\frac{a_\tau}{a_0} = e^{\frac{r_1-\rho}{\varepsilon}\tau} X(\tau)$, which yields a higher growth than the dynamics of the assets for type **V** women, $\frac{a_\tau}{a_0} = e^{\frac{r_1-\rho}{\varepsilon}\tau}$, as $X(\tau) \geq 1 \Leftrightarrow \varepsilon \geq 1$. ■

Lemma 1 *After the procreation attempt, the asset growth rate of type* **V** *women is larger than that of type* **M** *women if and only if*

$$2\varepsilon < 1 + \sqrt{\frac{8(r_1 - r_2) + \sigma^2}{\sigma^2}}. \tag{22}$$

[8] Estimations are made using three data sets: the Panel Study of Income Dynamics (PSID), the American Time Use Survey (ATUS), and the Consumer Expenditure Survey (CEX)

Proof: From Equation (9), the expected asset growth rate of type **M** women is given by: $\mathbb{E}\frac{a_t}{a_\tau} = e^{(r_2-q)(t-\tau)}$. The assets for type **V** women are, according to Equation (5): $\frac{a_t}{a_\tau} = e^{\frac{r_1-\rho}{\varepsilon}(t-\tau)}$. The latter is larger than the former if and only if (22) holds. ■

Lemma 2 *Delaying the date τ at which the woman tries having children generates more asset accumulation if the risk of sterility is ignored ($\pi = 1$). Accounting for the risk of sterility ($\pi = \pi(\tau) < 1$ with $\pi'(\tau) < 0$) reduces the effect and can even reverse it.*

Proof: It can be shown that: $\frac{\partial X(\tau)}{\partial \tau}|_{\pi=1} = p - q > 0$ and $\frac{\partial X(\tau)}{\partial \tau} = p - q + Z(\tau)$, with

$$Z(\tau) = \frac{e^{p\tau} - 1}{p} \left[\frac{\pi'(\tau)}{\varepsilon} \left(q^{-\epsilon} - p^{-\epsilon} \right) \right] s(\tau)^{1+\varepsilon} < 0.$$

■

The role of the procreation option is further highlighted by the dynamics of the assets of type **C** women:

$$a_t = a_0\, e^{\frac{r_1-\rho}{\varepsilon}\, t} + a_0 \left(e^{\frac{r_1}{\varepsilon}\, t} - e^{\frac{r_1-\rho}{\varepsilon}\, t} \right) \left(1 + \frac{X(\tau)s(\tau)}{p} \right)$$

The first term represents asset accumulation in the absence of procreation option. The second term is positive as $\varepsilon > 1$, again reflecting the idea that candidate mothers save more due to their expected future loss of income.

3.2 Age at Birth

After having derived the above results concerning asset growth, we now turn our attention to the procreation choice. The implicit expression for the optimal procreation attempt date is obtained from Equation (16):

$$\left(\frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} + c_\tau^{-\varepsilon} \left(r_1\, a_\tau - c_\tau \right) \right) e^{-\rho\tau} \;\; - \rho\varphi(\tau, a_\tau) \;\; + \pi'(\tau) \left(\left[q^{-\varepsilon} - p^{-\varepsilon} \right] \frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega \right) e^{-\rho\tau} = 0. \quad (23)$$

The first term represents the utility derived from the monetary gain of remaining childless a little longer. The second term is the cost of postponing the pleasure of having children. The last term represents the cost linked to the drop in fecundity induced by postponing ($\pi'(\tau) < 0$).

Proposition 2 *A high enough uncertainty leads to birth postponement:*

- ◇ *For $r_2 \geq r_1$, $\omega > 0$ and $\sigma = 0$, having a child has no cost. $\tau^* = 0$ i.e. it is then optimal to attempt to get pregnant as soon as possible.*

◇ *For* $r_2 = r_1$*, there exists a value* $\underline{\sigma} > 0$ *such that* $\sigma > \underline{\sigma} \Leftrightarrow \tau^* > 0$*, i.e. it is optimal to postpone birth.*

◇ *For* $r_2 = r_1$*, there exists a value* $\bar{\sigma} \geq 0$ *such that* $\sigma > \bar{\sigma} \Leftrightarrow \tau^* > t^m$

Proof: see Appendix C.1 ■

Birth irreversibility matters in this program because, as stated in **?**), there is a bad-news principle at work here: if future asset turns out to be less than expected, it is not possible for the woman to adjust and become childless. This possibility of regret appears if $W_1(t) > W_2(t)$ for $t > \tau$ which translate into a condition on asset accumulation after birth.[9]

We can also compute the value function as:

$$W(a_0) = \frac{(a_0 s(\tau) X(\tau))^{1-\varepsilon}}{(1-\varepsilon)p}(1 - e^{-p\tau}) + \varphi(\tau, a_\tau) \equiv \Phi(\tau, a_0)$$

where a_τ is a function of τ and a_0 through Equation (18) and τ solves (23). Part of the value comes from the possibility of trying and giving birth. The value of having this possibility, which we call "value of giving birth" is derived by comparing the value function with and without the possibility of procreating:

$$\text{value of giving birth} = W(a_0) - W_1(a_0),$$

where $W_1(a_0)$ is obtained from Equation (12). $W(a_0) - W_1(a_0)$ gives the willingness to pay for a child.[10] This value can be decomposed into the value of immediately trying and giving birth and the value of having the option to try and give birth later. Note that there is no information accruing in time, meaning that this option value, which we call "option value of giving birth" corresponds to the "pure postponement value" defined by Mensink and Requate (2005), as opposed to the option value for receiving information or "quasi-option value", which is the concept developed by Arrow, Fisher, Hanemann, and Henry (see Arrow and Fisher (1974), Henry (1974), and Fisher and Hanemann (1987)). This pure postponement value is however part of the Dixit-Pindyck option (see Dixit (1992), Pindyck (1991), and Dixit and Pindyck (1994)) which is the sum of the pure postponement value and of the quasi-option value. The option value of giving birth can be derived by comparing the value of giving birth at the optimal

[9] $W_1(t) > W_2(t) \Leftrightarrow a_{2t} < \left[a_{1t}^{1-\varepsilon} - \omega p^{\varepsilon}(1-\varepsilon)\right]^{\frac{1}{1-\varepsilon}} (q/p)^{\frac{\varepsilon}{1-\varepsilon}}$

[10] Note that Córdoba and Ripoll (2016) refer to this value as to the "option value of having a child" in a context where there is no timing decision, while we keep the term "option" for the additional value given by being able to choose the date of the birth.

date and the value of an immediate attempt to become a mother:

$$\text{option value of giving birth} = \text{value of giving birth} - \pi(0)W_2(a_0),$$

where $W_2(a_0)$ is obtained from Equation (8).

Instead of computing the total value of postponement (which corresponds to the option value of giving birth), one can also compute an instantaneous value of postponement at time t, which is obtained by computing the marginal value of postponing the birth attempt:

$$\text{marginal value of birth postponement} = \frac{\partial \Phi(t, a_0)}{\partial t}.$$

It is positive for all t lower than the optimal τ.

3.3 Childlessness

The model embeds three concepts of childlessness. When $\tau = +\infty$, the woman has never tried to have children. This resembles demographers' notion of voluntary childlessness, or the idea of opportunity-driven childlessness of Baudin, de la Croix, and Gobbi (2015). When $\tau = 0$ but $\theta = +\infty$, the woman wanted to have children at the beginning of the period considered, but could not. This is close to demographers' notion of involuntary childlessness, and the idea of natural sterility. When $\tau > 0$ but $\theta = +\infty$, the woman tried at some point in time to have children, but failed. This type of childlessness has an involuntary component, but also a voluntary one since, by postponing birth, the woman accepted a lower probability $\pi(\tau)$ of being fertile.

Proposition 3 *If $\rho < r_1$, there exists a unique level $\bar{\omega}$ of the lump-sum utility of having children such that for $\omega \leq \bar{\omega}$ the optimal age to try to have children is equal to or higher than menopause T, leading to type* **V** *women. There also exists a unique level $\tilde{\omega}$ of the lump-sum utility of having children such that for $\omega \geq \tilde{\omega}$ it is optimal to try to have children immediately (at 0). These two levels are such that $\bar{\omega} < \tilde{\omega}$.*

Proof: See Appendix C.2 ■

This proposition will allow us to calibrate the mean of parameter ω to match the observed childlessness rate.

4 Quantitative Analysis

In this section, we address four questions. First, does the income process (3) really differ between mothers and childless women, both in terms of growth and uncertainty? Second, can these differences in income explain why educated women delay having their first child and why more of them remain permanently childless? Third, what is the effect of exogenous shocks on these choices, including the effect of a hypothetical insurance mechanism for mothers and of free and efficient medically assisted reproduction technologies? Finally, how robust are the results to different choices of the subjective time discount rate and the relative risk-aversion parameter?

4.1 Identification of the Parameters

Table 1 summarizes our calibration strategy. Two parameters are set *a priori*. The subjective time discount rate ρ is set at 2% on an annual basis. The coefficient of relative risk aversion ϵ is set to 6. As we consider a CRRA instantaneous utility function, parameter ε represents both the relative risk aversion and the inverse of the intertemporal elasticity of substitution. In general, the literature favors a relative risk aversion coefficient less than 10 (see Gollier (2001)).[11] For example, using the Panel Study of Income Dynamics (PSID) for the years 1968-1997, French (2005) estimates the coefficient of relative risk aversion for men to be in the 2.2-5.1 range (depending on the specification). The identification comes both from the saving behavior according to which risk-averse agents save more in order to buffer themselves against the future, and from the labor supply since more risk-averse individuals work more hours when young in order to accumulate a buffer stock of assets for insurance against bad wage shocks when old. While French's estimates are about men, little has been done concerning women specifically, but the common result from experimental studies is that women are even more risk averse than men (Croson and Gneezy 2009). Finally, although there is no consensus concerning the value of the intertemporal elasticity of substitution, it is largely admitted that it should be less than unity. Our model shares similarities with a portfolio choice model which leads to very high values for risk aversion when brought to the data (Jorion and Giovannini (1993), Kocherlakota (1996), and Hansen et al. (2007)). Therefore, the value we have assigned to ε is a non-controversial upper bound for the relative risk aversion which is consistent with the model we use.

Date 0 in the model is assumed to represent age 18 in the data. Function $\pi(\cdot)$ is a generalization of the logistic function whose parameters are set to match the percentage of women who conceive

[11] With $\varepsilon = 10$, a household owning \$1M and facing a lottery that involves gaining or losing \$0.5M with equal probability is ready to give up \$0.46M or less to avoid the lottery: $\frac{(1-0.46)^{-9}}{-9} = 0.5\frac{(1+0.5)^{-9}}{-9} + 0.5\frac{(1-0.5)^{-9}}{-9}$.

	Parameter	value	target
ρ	subjective time discount rate	2%	fixed *a priori*
ϵ	relative risk aversion	6	fixed *a priori*
$\pi(t)$	success rate of pregnancy attempt	$\frac{0.96\exp(3.5-0.33t)}{0.012+\exp(3.5-0.33t)}$	from Léridon (2005)
r_1	return on assets when childless	Table 3	income growth – NLSY79
r_2	return on assets when mothers	Table 3	income growth – NLSY79
σ	std. dev. of Wiener process	Table 3	income range – NLSY79
m_ω	mean of the distribution of ω	2.143	mean age 1^{st} birth (cat. (7)) – NLSY79
s_ω	std. dev. of the distribution of ω	2.450	childlessness rate (cat. (7)) – NLSY79

Table 1: Identification of Deep Parameters - Summary

naturally after having started trying to get pregnant (lines b and g of Table I in Léridon (2005)). In practice, we assume:

$$\pi(t) = \begin{cases} \dfrac{a\exp(b-ct)}{d+\exp(b-ct)} & \text{if } t < T \\ 0 & \text{if } t >= T \end{cases}$$

We set $T = 35$ (i.e. 53 years). We set a, b, c, d to minimize

$$(\pi(12) - 0.921)^2 + (\pi(15) - 0.887)^2 + (\pi(17) - 0.846)^2 \\ + (\pi(19) - 0.782)^2 + (\pi(22) - 0.639)^2 + (\pi(24) - 0.489)^2 + (\pi(29) - 0.095)^2$$

subject to $\pi(0) = 0.96$ (we impose a natural sterility rate of 4%, see Baudin, de la Croix, and Gobbi (2015)). This gives $a = 0.96, b = 5.53, c = 0.33, d = 0.012$.

To calibrate the remaining parameters, we use data from the National Longitudinal Survey of Youth 1979 (NLSY79), which is a longitudinal project that follows the lives of a sample of American youth born between 1957-64. The eligible sample contains 9,964 respondents for whom data are available from Round 1 (1979 survey year) to Round 25 (2012 survey year), about half of them being women. We divide the sample into eight education categories, depending on the highest grade completed as of May 1994. Table 2 gives the mean age at first birth, its standard deviation, the percentage of women remaining permanently childless, and the percentage of ever married in the sample. The age at first birth and the childlessness rate are computed from the "number of children ever born" and "date of birth of first child" variables from XRND, which is a cross-round version of these variables (including information from June 1969 to December 2012).

The sample includes all women who actually have some income, independently from their marital status. An alternative is to consider married women only, which is coherent with the model when interpreted as a unitary model of the couple. A selection bias may arise here, because married women are not drawn randomly from the pool of women.[12] Another difficulty is that there is little evidence in the literature that the income and assets of couples is affected by childbearing as much as those of women (this is in line with the findings of Lundborg, Plug, and Rasmussen (2016)). In Section 4.5, we look at the robustness of the result to this selection criterion.

Not surprisingly, we observe a positive education gradient for both the mean age at first birth and the childlessness rate, with the age at first birth going from 18.2 to 28.7 when climbing up the education ladder, and childlessness rates going from 8.8% to 31.3%. We retrieve the result of Baudin, de la Croix, and Gobbi (2015), according to whom childlessness is U-shaped in education. The negative part of the U is obtained for low education levels. We also see in Table 2 that the variability (standard deviation) in the age at first birth is lower for the extreme categories.

Education category	Numb. of observ.	Mean years of education	Age at first birth Mean	Age at first birth Std. dev.	% childless	% married
Low education (1)	251	7.77	18.24	3.80	8.76	82.07
Less than high school (2)	300	10.52	19.34	4.13	7.00	78.00
High school compl. (3)	1868	12	21.70	4.98	12.15	84.42
Some college (4)	454	13	22.44	5.67	14.1	85.46
Some college (5)	469	14	24.38	5.45	20.04	83.16
Some college (6)	248	15	25.28	5.86	20.56	82.66
College completed (7)	551	16	27.64	5.08	24.32	87.66
More than college (8)	336	17.94	28.71	5.25	31.25	82.74
All	4477	13.08	22.93	5.79	16.04	84.01

Table 2: Education Groups, Age at First Birth, and Childlessness

The very high childlessness rate of the two top education categories is worth to be noted. Are these high rates the result of an early choice not to have children or, instead, comes as the outcome of a risky gamble (postponement)? Once calibrated, our model will be able to propose an answer to this question.

[12]The last column of Table 2 shows that the marriage rate (women who are or were married) is hump shaped in education - as in Baudin, de la Croix, and Gobbi (2015) who provide a quantitative analysis of this pattern.

To measure an individual's income, we sum farm and business income,[13] wages and salaries,[14] unemployment compensations received and other welfare payments. Before calculating the sum, we perform two transformations: we replace NA by 0 for farm and business income if wages and salaries are known, and replace NA by 0 for wages and salaries if farm and business income is known. Finally, we convert the income of various years into real income by dividing by the consumer price index. To bring Proposition 1 to the data, we ideally want to capture income growth after the decision to have children has been made. However, this is not possible, because the women in the sample are not old enough. As an approximation, we measure the growth rate of income between ages 39 and 45. Most women had their first child before age 39 (99.3%). Income at age 45 is taken as an average of income over three years (42-44-46 or 43-45-47 depending on age in 1979) to smooth business cycle effects. In case of missing data, the average is computed on the available one(s). Income at 39 is also taken as an average of three years.

Figure 1 plots kernel density estimations of income growth for each education category. Solid lines correspond to childless women and dashed lines to mothers. Compared with Table 2, we have lost some women because income is not observable for all of them. Let us stress three features that emerge from Figure 1. (1) For mothers, the mode of the distribution does not depend on education and is systematically higher than that for childless women. This reflects the fact that the income growth of mothers is systematically higher than that of childless women.[15] This is not inconsistent with $r_2 < r_1$, as shown in Lemma 1. (2) For childless women, the mode of the distribution moves rightwards as education increases, therefore catching up with the mode for mothers. (3) It appears clearly that the distribution is more dispersed for mothers than for childless women, reflecting the fact that the variance in the distribution of the growth in income is systematically higher for mothers than for childless women.[16] The latter result is in line with our idea that motherhood increases income risk.

To reduce the sensitivity to outliers, we run quantile regressions to measure the effect of education on the distribution of income growth, and infer the parameters from those quantiles. Table 3 presents the regression results. The independent variables include years of education, a dummy variable indicating if the woman is or has been married, a dummy variable indicating whether the women is separated, divorced or widowed at age 39, race fixed effects, and year of

[13]From question: How much did you receive after expenses [from your farms and businesses or professional practices/from your businesses or professional practices]?

[14]From question: How much did you receive from wages, salary, commissions, or tips from all (other) jobs, before deductions for taxes or anything else?

[15]This striking results holds when restricting the sample to married women; it is thus not related to the possible positive role of having a husband. It also holds when measuring income growth from 30 to 45.

[16]All these results remain true when restricting the sample of childless women to singles, who are less likely to have plans to give birth in the future.

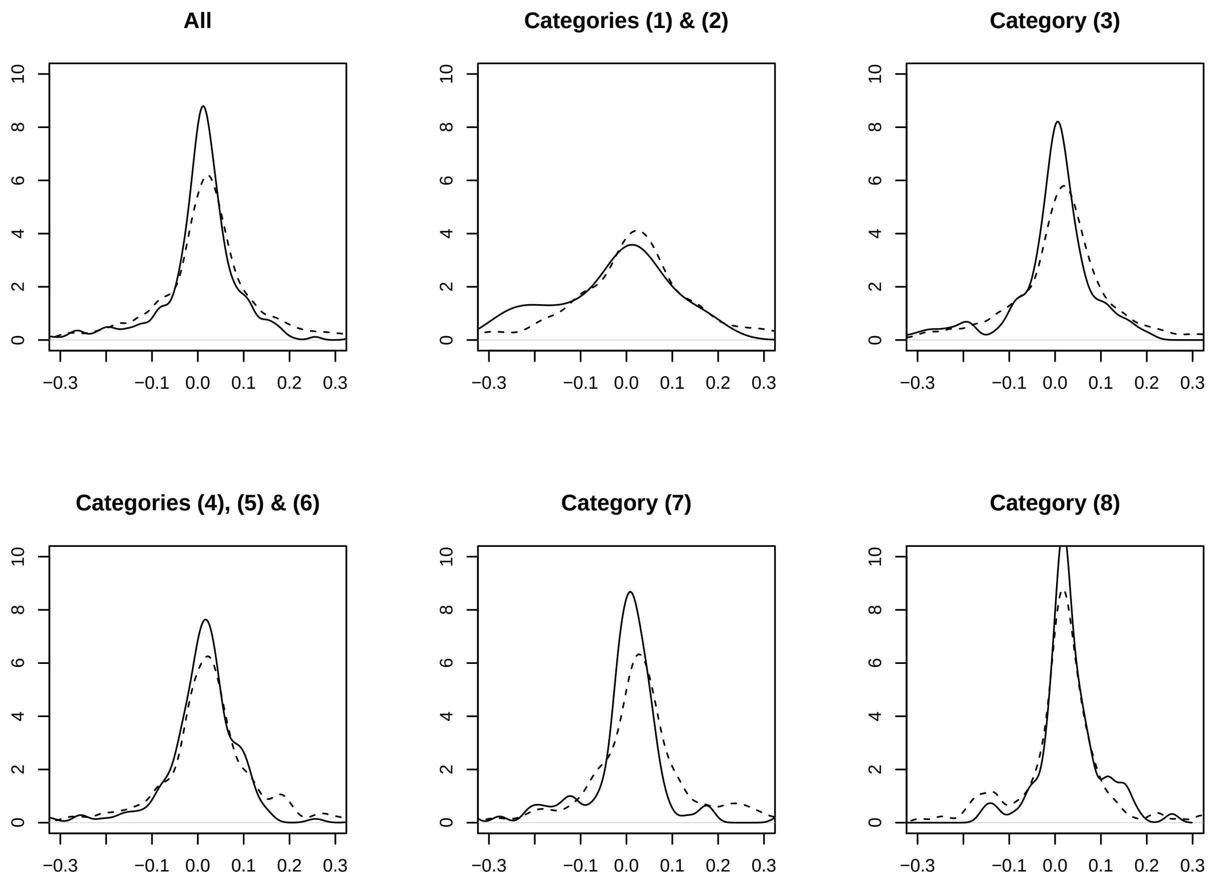


Figure 1: Kernel Density Estimations of Income Growth Distribution by Education Category. Childless Women (solid) and Mothers (dashed)

birth fixed effects. The reference category is a white single woman born in 1957. The results indicate that, for childless women, the median growth rate of income increases with education ($+0.0024^{***}$ per additional year of education). This is not true for mothers. For both groups, however, education helps to reduce the occurrence of bad outcomes, as can be seen from the determinants of $Q(0.07)$. This "protecting" effect of education is stronger for childless women than for mothers, and more statistically significant, (0.0191^{***} instead of 0.0052), reflecting the fact that having children increases uncertainty, especially for the highly educated.

Estimators for the growth rate of income for childless women, $\hat{g}_1$, and for mothers, $\hat{g}_2$, can be obtained using the fitted equation for the median ($Q(0.50)$) where the dummy "mother" is set to 0 and to 1 respectively. For the standard error of the distributions, $\hat{\sigma}_1$ and $\hat{\sigma}_2$, an estimator is given by taking the 7% trimmed range (the difference between the 7th and 93rd percentiles, $Q(0.93) - Q(0.07)$) and dividing by 3 (corresponding to 86% of the data of a normal distribution falling within 1.5 standard deviations of the mean). As in the model we

	Dependent variable: income growth between 39 and 45			
	OLS	$Q(0.07)$	$Q(0.50)$	$Q(0.93)$
Mothers				
Constant	0.0720^{***}	-0.1891^{***}	0.0481^{***}	0.4757^{***}
	(0.0265)	(0.0580)	(0.0152)	(0.0652)
years of educ.	0.0005	0.0052	0.0000	-0.0063^{*}
	(0.0013)	(0.0034)	(0.0006)	(0.0034)
Observations	2,705	2,705	2,705	2,705
Childless women				
Constant	-0.0834^{*}	-0.5680^{***}	0.0259	0.1131^{*}
	(0.0491)	(0.0902)	(0.0429)	(0.0654)
years of educ.	0.0056^{***}	0.0191^{***}	0.0024^{***}	-0.00001
	(0.0020)	(0.0051)	(0.0009)	(0.0038)
Observations	530	530	530	530

Note: *p<0.1; **p<0.05; ***p<0.01. All regressions include a married fixed effect, race fixed effects and year of birth fixed effects.

Table 3: Quantile Regression

Education	$\hat{g}_2$	$\hat{\sigma}_2^2$	$\hat{g}_1$	$\hat{\sigma}_1^2$	$\sigma = \sqrt{\hat{\sigma}_2^2 - \hat{\sigma}_1^2}$	r_2	r_1
1	0.0212	0.01935	-0.00226	0.01807	0.036	0.128	0.006
2	0.0212	0.01652	0.00435	0.01366	0.053	0.104	0.046
3	0.0212	0.01511	0.00786	0.01157	0.059	0.094	0.067
4	0.0212	0.01418	0.01026	0.01024	0.063	0.088	0.082
5	0.0212	0.01328	0.01266	0.00899	0.065	0.083	0.096
6	0.0212	0.01241	0.01506	0.00782	0.068	0.078	0.110
7	0.0212	0.01157	0.01746	0.00674	0.069	0.075	0.124
8	0.0212	0.01004	0.02206	0.00488	0.072	0.070	0.152

Table 4: Moments to Match and Calibration of r_1, r_2, and σ

have assumed no uncertainty for the childless, we compute σ^2 of the model as $\hat{\sigma}_2^2 - \hat{\sigma}_1^2$. Formally, knowing that the growth rate of income is equal to the growth rate of assets in all cases, we can rely on Equations (9), (11), and (13), and establish the following relations:

$$\sigma^2 = \hat{\sigma}_2^2 - \hat{\sigma}_1^2 \tag{24}$$

$$r_1 - p = \hat{g}_1 \quad \rightarrow \quad r_1 = \varepsilon \hat{g}_1 + \rho \tag{25}$$

$$r_2 - q = \hat{g}_2 \quad \rightarrow \quad r_2 = \varepsilon \hat{g}_2 + \rho - \frac{\varepsilon(\varepsilon - 1)\sigma^2}{2} \tag{26}$$

Notice here that σ^2 is the variance of the growth rate of assets over time taken after one period. It is measured with the variance across individuals, each individual being considered as one possible realization of shocks.

The above method allows to derive r_1, r_2, and σ for the whole sample, but also specific values for each education group. These are obtained by setting the "years of education" variable at its group mean when computing the quantiles to be matched. Table 4 summarizes the values of the moments to match, $\hat{g}_2$, $\hat{g}_1$, and $\hat{\sigma}_2^2 - \hat{\sigma}_1^2$, and the corresponding r_1, r_2, and σ_2.

We now have to set ω and a_0. As can be shown using Equation (23)(or seen from Equation (31) in the appendix), what matters is in fact $\omega a_0^{\varepsilon-1}$, showing that a_0 acts as a scaling factor for ω. We set $a_0 = 20$. Given the above parameters, we can compute the two thresholds of Proposition 3. The $\bar{\omega}$ such that all women with $\omega < \bar{\omega}$ are voluntarily childless is equal to 0.07. The $\tilde{\omega}$ such that all women with $\omega > \bar{\omega}$ attempt to have children at $t = 0$ is equal to 1.74. We now assume that ω is distributed across the population according to

$$\omega \sim \mathcal{N}(m_\omega, s_\omega^2).$$

The two parameters of the normal distribution function are set to match the mean age at first birth and the childlessness rate of the education category (7), as it is not in a corner regime with $\tau = 18$. They are equal to 27.64 years and 24.32% (from Table 2). This procedure allows to get these two levels right, but does not impose anything on the education gradient of the two variables. In the maximization problem of the woman, we impose the additional restriction that she cannot try to have children while at school; this requires $\tau > 6 + 16 + 1 - 18 = 5$ as school starts at 6, pregnancy requires (about) one year and 18 is time zero in our model. It yields $m_\omega = 2.143$ and $s_\omega = 2.450$.

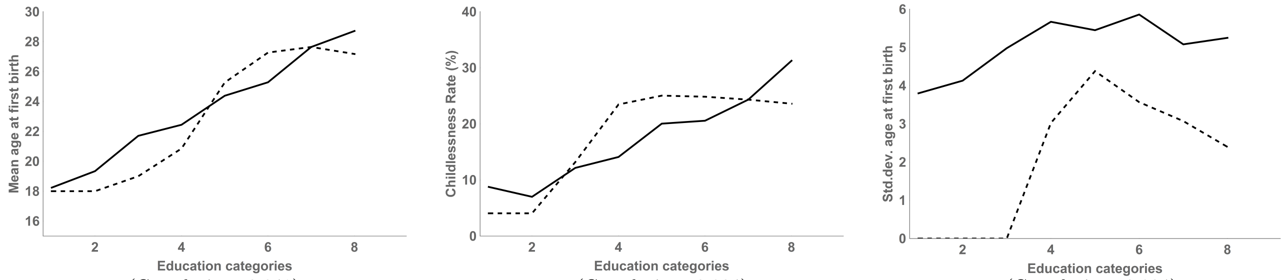
4.2 Overidentifying Restrictions

All the parameters of the model have now either been fixed a priori, or exactly identified with some moments computed from the NLSY79. None of them has been set so as to match the fact that both the age at first birth and the childlessness rate are increasing in education. We can therefore evaluate our model against these two facts. This is in line with the spirit of testing overidentifying restrictions, although there is no formal testing here as we do not do any statistical inference.

For each education group, we set the income process using the corresponding parameters from Table 4. Next, we generate an artificial population with a taste for children ω drawn from its normal distribution. We suppose ω is drawn from the same distribution for all education categories, otherwise is would be straightforward to match education-specific moments with education-specific preference parameters. We impose that each woman in this population cannot bear children while at school, and compute the optimal age for a pregnancy attempt, τ, and the childlessness probability given by $1 - \pi(\tau)$. Finally, we average these two numbers across women. The results are shown in Figure 2 for the eight education categories. The sign of the education gradient is correct for both the age at first birth and the childlessness rate. The size of the gradient is underestimated for childlessness (middle panel), but less so for the age at first birth (left panel). The model tends to underestimate both the age at first birth and the childlessness rate for the highest education category, reflecting that other considerations than income may play a role for this category.

We also checked the predictions of the model for the standard deviation of the age at first birth. The right panel of Figure 2 shows that the level of the standard error is systematically underestimated, but its hump-shaped pattern is well reproduced. This latter result is explained by the fact that the extreme categories of education more often hit the bounds of the set of possible ages at pregnancy, hence lowering variability. The underestimation of the standard deviation in the age at first birth comes from the fact that we have neglected other sources of variability, for instance when we assume that a birth, if any, immediately follows the pregnancy attempt.

Figure 2 illustrates the new mechanism we have put forward in this paper. Motherhood increases income volatility (Proposition 2), in particular for highly educated women. This is why they prefer to postpone birth, in order to accumulate enough assets before being hit by the possibly negative shocks related to having a child. This is also why more of them opt for permanent childlessness voluntarily.



(Correlation=0.941) (Correlation=0.804) (Correlation=0.824)

Figure 2: Overidentifying Restrictions: Education Gradient - data:solid, simulated: dashed

4.3 The Roots of Childlessness and Policy Analysis

The model also allows to decompose childlessness into the three parts mentioned in Section 3: voluntary childlessness includes those who never try to have children; natural sterility includes (a) those who wanted to have children at the beginning of the period considered, but could not, and (b) those who tried later on and could not because of benchmark sterility $\pi(0)$; and postponement childlessness includes those who tried at some date $\tau > 0$ to have children, but failed because of increased sterility at τ, $\pi(\tau) - \pi(0) > 0$. Consider the two education classes for which simulated childlessness is very close to observed chidlessness: High school completed (3) and College completed (7). For the High school completed, the total simulated childlessness rate of 13.18% includes 9.02% of voluntary childlessness, 3.52% of natural sterility, and 0.16% of postponement childlessness, and 0.48% of sterile women not wanting children. For the College completed, the total simulated childlessness rate of 24.32% includes 18.57% of voluntary childlessness, 3.07% of natural sterility, and 1.75% of postponement childlessness,[17] and 0.93% of sterile women not wanting children.

We now simulate the effect of two exogenous changes on women's behavior. The first change, labelled "full insurance", consists in the disappearance of the excess volatility undergone by mothers. Technically, we set $\sigma = 0$. This means that mothers now face the same uncertainty as childless women. Table 5 shows the results. The full insurance scenario drastically reduces the mean age at first birth for education categories 5 and up. It also reduces the childlessness rate for these categories by 1 to 2 percentage points. This policy operates by incentivizing low ω women to try to have children, whereas they would opt for being voluntarily childless otherwise. This result again stresses the importance of the additional uncertainty undergone by mothers for their procreation decision. Our results echo those of Lalive and Zweimüller (2009), who show, in the case of Austrian reforms, that "both cash transfers and job protection are relevant" to increase fertility (in their case, going from one to two children).

[17] One can guess that this figure would be even higher for more recent data.

Steps towards full insurance include a social security policy reducing mothers' lack of income security, in the spirit of the parental leaves with job protection which have already been implemented in various ways in some OECD countries but (nearly) not in the US. Using the model of section 2, one can compute the wealth transfer to be received at motherhood that would compensate the effect of uncertainty on the value and the birth timing.[18] As uncertainty directly affects the post-birth value of mothers, such a transfer T should be designed as an equivalent variation in a_τ such that

$$W_2(a_\tau + T) = W_2(a_\tau)_{\sigma=0}.$$

Therefore,

$$T = a_\tau((q/q_{\sigma=0})^{\frac{\varepsilon}{1-\varepsilon}} - 1).$$

We can compute the value of T for the various education groups. Normalizing the transfer in favor of the lowest education group to 1, the transfer which neutralizes the effect of uncertainty would be equal to 2.89 for women with less than high school, 4.08 for high school graduates, 7.65 for college graduates, and 9.03 for the highest group with more than college. Such a full insurance transfer would thus be strongly anti-redistributive.

The second "policy" we implement consists in very strong assisted procreation techniques, which amount to making candidate mothers 3 years younger, i.e. the new $\pi^{\text{new}}(t) = \pi(t-3)$. Such a "rejuvenation" affects childlessness negatively by allowing older parents to have children. Making people younger also has an "incentive" effect: all the categories 4 and above delay the birth of their first child by a little less than one year. The overall effect on childlessness is stronger than that of the previous policy for the extreme education categories only. For the middle categories, the incentive to delay birth is stronger (because these categories include fewer persons in the corner regimes $\tau^\star = 0$ and $\tau^\star = t^m$), and the effect on childlessness is similar to the one generated by the full insurance policy.

4.4 Robustness to the Choice of Parameters

We now analyze how robust the above results are to different choices of the subjective time discount rate ρ and the relative risk aversion parameter ε. Let us first consider ε. Changing the value of ε affects the results in two very different ways. First, it affects the computation of the returns r_1 and r_2 as a function of the observed growth rates $\hat{g}_1$ and $\hat{g}_2$, and uncertainty σ^2. Let us call this effect a *recalibration effect*. Second, it affects the results by changing the women's preferences; it is a *behavioral effect*.

[18]However, it must be kept in mind that such a policy may create an incentive for mothers to leave the labor market, which we disregard in this study.

	education cat.	full insurance	assisted procreation
Δ age at first birth	1	0.00	0.00
	2	0.00	0.00
	3	0.00	0.20
	4	-0.85	0.61
	5	-3.33	0.88
	6	-3.56	0.89
	7	-3.00	0.81
	8	-1.47	0.66
Δ childlessness rate	1	0.00	-0.01
	2	0.00	-0.01
	3	-0.96	-1.03
	4	-2.59	-1.35
	5	-1.77	-1.70
	6	-1.38	-1.85
	7	-0.96	-1.86
	8	-0.66	-1.79

Table 5: Effect of Policy on Fertility Timing Choices, by Education Category

To assess the *recalibration effect*, one can use Equations (25)-(26) and see that reasonable values for r_1 and r_2 require relatively strict conditions on ε. For example, for college educated women, imposing that childless women enjoy a higher return than mothers ($r_1 > r_2$) but not by more than, say, 6% ($r_1 < r_2 + 0.06$) implies that ε should be between 2.55 and 6.42. Outside this interval, women will either always want to have a child immediately (when r_1 is close to r_2), or never want to have children (when $r_1 - r_2$ is large).

Keeping r_1 and r_2 at their benchmark values, we can analyze the *behavioral effect* of changing ρ and ε on various outcomes. Table 6 provides the results. Given a range of values for ρ and ε (first two columns), the table shows the three correlations between actual and simulated values when the level of education varies. For the benchmark, in bold, the correlations summarize the information given in Figure 2. The "fit" of the mean age at birth remains good for all the parameters considered. The "fit" of the childlessness rate also remains what it is in the benchmark (good but misses the target for the highly educated women). The last four columns of Table 6 show the main effect of the policies considered for college educated women. The sixth column shows that the size of the drop of about 3 years in the age at first birth following the removal of additional uncertainty linked to motherhood is very robust (but when $\varepsilon = 4$). The drop in the childlessness rate however depends on the parameters. The last two columns show that the size of the effect of medically assisted procreation, which is increasing by less than 1 year(s) the age at first birth and decreasing the childlessness rate by 1.86%, is quite consistent across parametrizations.

Parameters		Overidentifying tests			Policy: $\sigma = 0$,		$\pi^{\text{new}}(t) = \pi(t-3)$	
ρ	ε	corr(E τ)	corr(cln)	corr(std. τ)	$\Delta\ \tau$	Δ cln	$\Delta\ \tau$	Δ cln
0.02	4	0.96	0.83	0.75	-1.98	-0.99	+0.96	-1.95
0.02	5	0.95	0.82	0.83	-2.49	-1.14	+0.91	-2.04
0.02	**6**	**0.94**	**0.80**	**0.82**	**-3.00**	**-0.96**	**+0.81**	**-1.86**
0.02	7	0.94	0.76	0.86	-3.41	-1.10	+0.77	-1.96
0.01	6	0.94	0.79	0.84	-2.93	-0.97	+1.01	-1.91
0.02	**6**	**0.94**	**0.80**	**0.82**	**-3.00**	**-0.96**	**+0.81**	**-1.86**
0.04	6	0.94	0.79	0.82	-2.98	-1.70	+0.73	-2.18
0.06	6	0.94	0.79	0.84	-2.93	-0.97	+0.72	-2.54

Note: 'cln' = childlessness rate. Effects of policy are reported for college educated women (education group 7)

Table 6: Effect of Changing ρ and ε on Fit and Policy

4.5 Robustness to Sample Selection and to Additional Controls

In the benchmark analysis, we have used the sample of all women with a positive individual income to deduce the parameters r_1, r_2, and σ from the distribution of income growth across women. In this subsection, we first consider an alternative sample, that of women who are or have been married. In a context where marriage acts as an insurance against risk, it is indeed interesting to see whether the marital status matters for our estimation.[19] This reduces the sample by 16% but disproportionately affects the extreme education categories (less than high school and more than college). The mean age at first birth is almost unaffected by this selection, but the childlessness rate is reduced from 16.04% to 11.94%. Concerning the income processes examined through the lens of the quantile regression, this "protecting" effect of education, which was 0.0191*** for childless women and 0.0052 for mothers in the full sample, is reduced to 0.0107* for childless women and remains the same at 0.0059* for mothers. It implies that the loss associated with being a mother is reduced when we consider married women only: for

[19] As an alternative to reduce the sample to married women, we have also looked for including the income of the partner in the quantile regressions. Unfortunately, data limitations prevent us from doing so. We observe partner income is a relatively small number of cases (1496 mothers and 197 childless women), and we are reluctant in interpreting the missing values as zeros.

the highest education category, $r_2 = 11.8\%$ and $r_1 = 15.3\%$, while they were given by $r_2 = 7\%$ and $r_1 = 15.2\%$ on the full sample.

The second robustness analysis is designed to address the issue of reverse causality between parenthood and years of schooling. In the sample, some women might have decided to stop schooling after their first child, rather than to postpone childbearing until they had completed their education, i.e. for given r_1, r_2, and σ. We accordingly remove all women who had children before the age of 16 from the sample (16 marks the end of compulsory schooling in most US states during the period considered; see Appendix 2 in Angrist and Krueger (1991)). This reduces the sample by 3.5%, but more so in the low-education category. In the selected sample, the average age at first birth increases to 23.33 instead of 22.93 in the full sample. Childlessness also mechanically increases to 16.68%, as young mothers are removed from the sample. The coefficients of the quantile regression are very similar to the benchmark.

Figures 3 and 4 in Appendix plot kernel density estimations of income growth for each education category – to be compared with Figure 1 for the full sample. Solid lines correspond to childless women and dashed lines to mothers. It remains true in both smaller samples that the variance of income growth is larger for mothers than for childless women. This is confirmed by the (non-reported) estimations of the same quantile regressions.

Sample	Nobs	Overidentifying tests			Policy: $\sigma = 0$,		$\pi^{\text{new}}(t) = \pi(t-3)$	
		corr(E τ)	corr(cln)	corr(std. τ)	$\Delta\ \tau$	Δ cln	$\Delta\ \tau$	Δ cln
All	**4477**	**0.94**	**0.80**	**0.82**	**-3.00**	**-0.96**	**+0.81**	**-1.86**
Married	3761	0.94	0.89	0.59	-2.90	-1.83	+0.88	-1.67
No teenage mother	4304	0.90	0.74	0.84	-3.09	-1.12	+0.83	-1.33
Controlling # kids	4477	0.93	0.87	0.65	-2.44	-1.27	+0.95	-1.96

Note: 'cln' = childlessness rate

Table 7: Effect of Changing Sample on Fit and Policy

The results of the simulations are presented in Table 7. In a nutshell, reducing the sample to either married women or mothers with children above 15 slightly worsens the fit of the model. It is as if single women (either childless or mothers) and teenage mothers were part of the story we tell, and help the model fit the facts. Abstracting from them however does not really affect the size of the effects of the insurance policy and of the medically assisted procreation program.

Beyond sample selection issues, one may also want to assess how far the results are robust when one changes the set of control variables in the quantile regressions. So far we have neglected the intensive margin of fertility, as the model was only about a 0/1 choice. In the data, women may have more than one child, and we can control for it in the regression. Accordingly, we introduce the number kids as a control in the regression for mothers. The number of kids influences positively the growth rate of income at all quantiles, but more so for $Q(0.93)$, implying that people with say four kids have more uncertainty than those with two kids, but a higher expected growth rate. All in all, these estimation results translate into a slightly different calibration, and into different simulations results (last line of Table 7). The overall picture is not modified as the effects of policies are in general similar than in the benchmark. Still, we think an extension of the model to the higher order births (siblings) as well would be worth considering.

5 Conclusion

We know from the literature that the opportunity cost of having children is greater for highly educated women than for low-educated women. This leads the former to have fewer children or to be childless more often, creating a differential fertility between the extremes of the education spectrum (de la Croix and Doepke 2003, Vogl 2016). This paper highlights one important channel of this mechanism by relying on the analogy between postponing birth and delaying an irreversible investment.

We have seen from the National Longitudinal Survey of Youth 1979 that education protects against negative shocks to income. However, this protecting effect of education is stronger for childless women than for mothers. It is very clear from the data that having children increases income uncertainty, especially for the highly educated.

Facing this uncertainty, educated women expecting to have a child accumulate more assets, in order to prevent a decrease in the certainty-equivalent asset growth. For them, postponing birth has a value, the "option value of birth," which corresponds to a "pure postponement value" as defined by Mensik and Requate (2005).

Our approach also allows to precisely define a new notion of childlessness related to postponement. Some educated women will try to have children at some point, but a fraction of them will fail. This type of childlessness has an involuntary component, but also a voluntary one since, by postponing birth, women accept a lower probability of being fertile.

The calibration of the model shows that the income uncertainty aspect is paramount compared to the biological clock. Indeed, if mothers could be insured against the income risk of having children, the age at first birth for the more educated categories would drop very strongly.

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A Solving the Post-Birth Program

We note the variable part of the value function as $V(a_\tau) = W_2(a_\tau) - \omega$. The corresponding Bellman equation is

$$\begin{aligned}
V(a_t) &= \max_{c_t} \left\{ u(c_t)dt + e^{-\rho\, dt} E\left[V(a_{t+dt})\right] \right\} \\
&\Leftrightarrow\ V(a_t) = \max_{c_t} \left\{ u(c_t)dt + (1-\rho\, dt)\left(V(a_t) + E\left[dV(a_t)\right]\right) \right\} \\
&\Leftrightarrow\ \rho V(a_t)dt = \max_{c_t} \left\{ u(c_t)dt + (1-\rho\, dt) E\left[dV(a_t)\right] \right\}
\end{aligned}$$

using $e^{-\rho\, dt} \approx (1-\rho\, dt)$ for small dt. To solve for the value function, we make an educated guess, $V(a_t) = D_2 \frac{a_t^{1-\varepsilon}}{1-\varepsilon}$ where D_2 is a constant to be determined. According to Itô's lemma:

$$\begin{aligned}
E\left[dV(a_t)\right] &= \frac{\partial V(a_t)}{\partial a_t} E[da_t] + \frac{1}{2}\frac{\partial^2 V(a_t)}{\partial a_t^2} E[(da_t)^2] \\
&= D_2 a_t^{1-\varepsilon} \left(r_2 - \frac{c_t}{a_t} - \frac{\varepsilon}{2}\sigma_2^2 \right) dt
\end{aligned}$$

as $E[da_t] = (r_2 a_t - c_t)dt$ and $E[(da_t)^2] = \sigma_2^2 a_t^2 dt$ (as $(dt)^2 \approx 0$, $(dt)^{3/2} \approx 0$ and $E[(dz)^2] = dt$)

The Bellman equation then becomes:

$$\rho V(a_t) = \max_{c_t} \left\{ \frac{c_t^{1-\varepsilon}}{1-\varepsilon} + D_2 a_t^{1-\varepsilon} \left(r_2 - \frac{c_t}{a_t} - \frac{\varepsilon}{2}\sigma_2^2 \right) \right\}$$

The first-order condition with respect to consumption is: $c_t = D_2^{-1/\varepsilon} a_t$. In order to determine constant D_2, the Bellman equation can be rewritten:

$$\begin{aligned}
\rho D_2 \frac{a_t^{1-\varepsilon}}{1-\varepsilon} &= \frac{\left(D_2^{-1/\varepsilon} a_t\right)^{1-\varepsilon}}{1-\varepsilon} + D_2 a_t^{1-\varepsilon} \left(r_2 - D_2^{-1/\varepsilon} - \frac{\varepsilon}{2}\sigma_2^2 \right) \\
&\Leftrightarrow\ \frac{\rho}{1-\varepsilon} = \frac{D_2^{-1/\varepsilon}}{1-\varepsilon} + \left(r_2 - D_2^{-1/\varepsilon} - \frac{\varepsilon}{2}\sigma_2^2 \right) \\
&\Leftrightarrow\ \frac{\rho}{1-\varepsilon} = \frac{\varepsilon D_2^{-1/\varepsilon}}{1-\varepsilon} + \left(r_2 - \frac{\varepsilon}{2}\sigma_2^2 \right) \\
&\Leftrightarrow\ D_2 = \left[\frac{1}{\varepsilon} \left(\rho + (\varepsilon - 1)\left(r_2 - \frac{\varepsilon}{2}\sigma_2^2 \right) \right) \right]^{-\varepsilon} = q^{-\varepsilon},
\end{aligned}$$

where q is defined in Equation (7). Therefore

$$W_2(a_\tau) = q^{-\varepsilon} \frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega.$$

B Solving the Full Program

Standard integration by parts yields:

$$\int_0^\tau \mu_t \dot{a}_t dt = \mu_\tau a_\tau - \mu_0 a_0 - \int_0^\tau \dot{\mu}_t a_t \; dt,$$

which allows to rewrite $W(a_0)$ as:

$$W(a_0) = \int_0^\tau \left(H(c_t, a_t, \mu_t) + \dot{\mu}_t a_t\right) dt + \varphi(\tau, a_\tau) - \mu_\tau a_\tau + \mu_0 a_0.$$

The first-order variation of $W(a_0)$ with respect to the state and control variable's path for a given a_0 but for τ and a_τ free yields:

$$\begin{aligned} dW(a_0) \quad = \quad & \int_0^\tau \left(\frac{\partial H(c_t, a_t, \mu_t)}{\partial a_t} da_t + \frac{\partial H(c_t, a_t, \mu_t)}{\partial c_t} dc_t + \dot{\mu}_t \; da_t \right) dt \\ & + \left[H(c_\tau, a_\tau, \mu_\tau) + \dot{\mu}_\tau a_\tau\right] d\tau + \frac{\partial \varphi(\tau, a_\tau)}{\partial a_\tau} da_\tau + \frac{\partial \varphi(\tau, a_\tau)}{\partial \tau} d\tau \\ & - \dot{\mu}_\tau a_\tau d\tau - \mu_\tau da_\tau. \end{aligned}$$

Rearranging terms leads to:

$$\begin{aligned} dW(a_0) \quad = \quad & \int_0^\tau \left[\left(\frac{\partial H(c_t, a_t, \mu_t)}{\partial a_t} + \dot{\mu}_t \right) da_t + \frac{\partial H(c_t, a_t, \mu_t)}{\partial c_t} dc_t \right] dt \\ & + \left[H(c_\tau, a_\tau, \mu_\tau) + \frac{\partial \varphi(\tau, a_\tau)}{\partial \tau} \right] d\tau \\ & + \left[\frac{\partial \varphi(\tau, a_\tau)}{\partial a_\tau} - \mu_\tau \right] da_\tau. \end{aligned}$$

A trajectory is (locally) optimal if any (local) departure from it decreases the value function, that is $dW(a_0) \leq 0$ for any $da_t, t \in (0, \tau)$, for any $dc_t, t \in (0, \tau)$, and for any $d\tau$ and da_τ, which gives the following necessary conditions for an interior maximizer:

$$\begin{aligned} \frac{\partial H(c_t, a_t, \mu_t)}{\partial c_t} &= 0 \\ \frac{\partial H(c_t, a_t, \mu_t)}{\partial a_t} + \dot{\mu}_t &= 0 \end{aligned}$$

$$
\begin{aligned}
H(c_\tau, a_\tau, \mu_\tau) + \frac{\partial \varphi(\tau, a_\tau)}{\partial \tau} &= 0 \\
\frac{\partial \varphi(\tau, a_\tau)}{\partial a_\tau} - \mu_\tau &= 0
\end{aligned}
$$

The first two conditions are standard Pontryagin conditions. The last two conditions may be interpreted as optimality conditions with respect to the switching time τ and the free state value a_τ. The third one equalizes the marginal benefit of waiting to the marginal cost of waiting. The last one is a continuity condition: it implies that the shadow price of the state variable at the time of the switch, μ_τ, is equal to the expected marginal value of the state variable in τ (derived from the programs after the switch).

The two standard Pontryagin conditions imply:

$$
\begin{aligned}
/c_t \quad &: \quad u'(c_t)e^{-\rho t} = \mu_t \\
/a_t \quad &: \quad \dot{\mu}_t/\mu_t = -r_1 \Rightarrow \mu_t = \mu_0 e^{-r_1 t} \\
&\Rightarrow \quad \dot{c}_t/c_t = (r_1 - \rho)/\varepsilon \Rightarrow c_t = c_0 e^{\frac{r_1-\rho}{\varepsilon}t} \text{ and } c_t = \left(\mu_t e^{\rho t}\right)^{-1/\varepsilon}
\end{aligned}
$$

Therefore, the dynamics of assets can be rewritten as:

$$
\dot{a}_t = r_1 \, a_t - c_0 e^{\frac{r_1-\rho}{\varepsilon}t}
$$

Using a variable change $x_t = a_t e^{-r_1 t}$ we solve for $x_t = (c_0/p)e^{\left(\frac{r_1-\rho}{\varepsilon} - r_1\right)t} + \overline{x}$, where p is defined in Equation (6) and $\overline{x}$ is a constant to be determined. Therefore:

$$
a_t = \frac{c_0}{p} \, e^{\left(\frac{r_1-\rho}{\varepsilon}\right)t} + \overline{x} e^{r_1 t}
$$

Without the procreation option, the transversality condition would imply $\overline{x} = 0$ and c_0 would be determined by a_0.

Moreover at $t = 0$:

$$
\begin{aligned}
a_0 \quad &= \tfrac{c_0}{p} + \overline{x} \\
&\Leftrightarrow \qquad \overline{x} = a_0 - \frac{c_0}{p} \\
&\Rightarrow \qquad a_t = \frac{c_0}{p} e^{\left(\frac{r_1-\rho}{\varepsilon}\right)t} + \left[a_0 - \frac{c_0}{p}\right] e^{r_1 \, t}
\end{aligned}
$$

$$
a_t = a_0 e^{r_1 t} + \frac{c_0}{p} \left[e^{\left(\frac{r_1-\rho}{\varepsilon}\right)t} - e^{r_1 t} \right] \tag{27}
$$

The fourth condition allows to find a_τ as a function of τ:

$$\frac{\partial \varphi(\tau, a_\tau)}{\partial a_\tau} - \mu_\tau = 0$$

$$\begin{aligned}
&\Leftrightarrow \quad \mu_\tau = e^{-\rho\tau}\left[\pi(\tau)q^{-\varepsilon} + (1-\pi(\tau))p^{-\varepsilon}\right] a_\tau^{-\varepsilon} \\
&\Leftrightarrow \quad \mu_\tau = e^{-\rho\tau}(s(\tau)a_\tau)^{-\varepsilon} \\
&\Rightarrow \quad \mu_0 = \mu_\tau e^{r_1\tau} = e^{(r_1-\rho)\tau}(s(\tau)a_\tau)^{-\varepsilon}
\end{aligned}$$

where we have introduced the following notation:

$$s(t) = \left(\pi(t)q^{-\varepsilon} + (1-\pi(t))p^{-\varepsilon}\right)^{-1/\varepsilon}$$

which is valid in particular for $t = \tau$. This allows to identify c_0 as a function of τ and a_τ:

$$c_0 = (\mu_0)^{-1/\varepsilon} = \left(e^{(r_1-\rho)\tau}(s(\tau)a_\tau)^{-\varepsilon}\right)^{-1/\varepsilon} = \left(e^{(r_1-\rho)\tau}\right)^{-1/\varepsilon} s(\tau)a_\tau \tag{28}$$

Using (27) and (28), it is then possible to obtain an expression for a_τ as a function of τ and a_0:

$$a_\tau = \frac{e^{r_1\tau}}{1 + s(\tau)\left[e^{p\tau} - 1\right]/p} a_0 \tag{29}$$

Note that without the procreation option, this expression would become:

$$a_\tau = e^{\frac{r_1-\rho}{\varepsilon}\tau} a_0$$

Therefore, it is possible to express $X(\tau)$ the effect of the procreation option as follows:

$$a_\tau = e^{\frac{r_1-\rho}{\varepsilon}\tau} X(\tau) a_0$$

with

$$X(\tau) = \frac{e^{p\tau}}{1 + s(\tau)\left[e^{p\tau} - 1\right]/p}$$

$X(\tau) \geq 1$ if and only if $\varepsilon \geq 1$.
It is now possible to express the dynamics of a_t:

$$a_t = a_0 \left[e^{r_1 t} - \frac{e^{p\tau}\left[e^{r_1 t} - e^{\left(\frac{r_1-\rho}{\varepsilon}\right)t}\right]}{ps(\tau) + \left[e^{p\tau} - 1\right]} \right] \tag{30}$$

In addition, using $c_\tau = c_0 e^{\frac{r_1-\rho}{\varepsilon}\tau}$, Equations (28) and (30), it is possible to express consumption as a function of τ only:

$$c_\tau = \frac{e^{p\tau} s(\tau)}{1 + s(\tau)\left[e^{p\tau} - 1\right]/p} a_0 e^{\frac{r_1-\rho}{\varepsilon}\tau}$$

Note that absent the procreation option, this expression would become:

$$c_\tau = p e^{\frac{r_1-\rho}{\varepsilon}\tau} a_0$$

Finally, the third condition gives a second relation between a_τ and τ:

$$\begin{aligned}
& H(c_\tau, a_\tau, \mu_\tau) + \frac{\partial \varphi(\tau, a_\tau)}{\partial \tau} = 0 \\
\Leftrightarrow\ & \frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} e^{-\rho\tau} + \mu_\tau \left(r_1 a_\tau - c_\tau\right) - \rho\varphi(\tau, a_\tau) \\
& \qquad + e^{-\rho\tau}\left[\pi'(\tau)\left(q^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega\right) - \pi'(\tau)\left(p^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}\right)\right] = 0 \\
\Leftrightarrow\ & \frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} e^{-\rho\tau} + \mu_\tau \left(r_1 a_\tau - c_\tau\right) - \rho\varphi(\tau, a_\tau) \\
& \qquad + e^{-\rho\tau}\left[\pi'(\tau)\left(q^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + \omega\right) - \pi'(\tau)\left(p^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}\right)\right] = 0 \\
\Leftrightarrow\ & \frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} e^{-\rho\tau} + \mu_\tau \left(r_1 a_\tau - c_\tau\right) - \rho\varphi(\tau, a_\tau) \\
& \qquad + e^{-\rho\tau}\pi'(\tau)\left[q^{-\varepsilon} \ - p^{-\varepsilon}\right]\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + e^{-\rho\tau}\pi'(\tau)\omega = 0.
\end{aligned}$$

To obtain a function of τ only, recall that

$$\begin{aligned}
s(\tau)^{-\varepsilon} &= \pi(\tau) q^{-\varepsilon} + (1 - \pi(\tau)) p^{-\varepsilon} \\
&= \pi(\tau)(q^{-\varepsilon} - p^{-\varepsilon}) + p^{-\varepsilon} \\
&\Rightarrow \frac{s(\tau)^{-\varepsilon} - p^{-\varepsilon}}{\pi(\tau)} = q^{-\varepsilon} - p^{-\varepsilon}
\end{aligned}$$

and

$$c_\tau^{-\varepsilon} e^{-\rho\tau} = \mu_\tau$$

then

$$\begin{aligned}
& \frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} e^{-\rho\tau} + r_1 e^{-\rho\tau} s(\tau)^{-\varepsilon} a_\tau^{1-\varepsilon} - c_\tau^{1-\varepsilon} e^{-\rho\tau} - \rho\varphi(\tau, a_\tau) \\
& \qquad + e^{-\rho\tau}\frac{\pi'(\tau)}{\pi(\tau)}\left(g(\tau) - p^{-\varepsilon}\right)\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} + e^{-\rho\tau}\pi'(\tau)\omega = 0
\end{aligned}$$

$$
\begin{aligned}
\text{where } \varphi(\tau, a_\tau) &= e^{-\rho\tau}\left[\pi(\tau)\left(q^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}+\omega\right)+(1-\pi(\tau))p^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}\right] \\
&= e^{-\rho\tau}\left[\pi(t)q^{-\varepsilon}+(1-\pi(\tau))p^{-\varepsilon}\right]\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}+e^{-\rho\tau}\pi(\tau)\omega \\
&= e^{-\rho\tau}s(\tau)^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}+e^{-\rho\tau}\pi(\tau)\omega
\end{aligned}
$$

therefore

$$
\frac{\varepsilon c_\tau^{1-\varepsilon}}{1-\varepsilon}+r_1 g(\tau)a_\tau^{1-\varepsilon}-\rho\left[s(\tau)^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}+\pi(\tau)\omega\right]+\frac{\pi'(\tau)}{\pi(\tau)}\left(g(\tau)-p^{-\varepsilon}\right)\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}+\pi'(\tau)\omega=0
$$

$$
\Leftrightarrow \frac{\varepsilon c_\tau^{1-\varepsilon}}{1-\varepsilon}+r_1 s(\tau)^{-\varepsilon}a_\tau^{1-\varepsilon}-\rho\left[s(\tau)^{-\varepsilon}\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon}\right]+\frac{\pi'(\tau)}{\pi(\tau)}\left(s(\tau)^{-\varepsilon}-p^{-\varepsilon}\right)\frac{a_\tau^{1-\varepsilon}}{1-\varepsilon} = \left(\rho\pi(\tau)-\pi'(\tau)\right)\omega
$$

Recall

$$
c_\tau = c_0 e^{\frac{r_1-\rho}{\varepsilon}\tau} = \left(g(\tau)\right)^{-1/\varepsilon}a_\tau
$$

Therefore:

$$
a_\tau^{1-\varepsilon}\left(\frac{\varepsilon\left(s(\tau)\right)^{1-\varepsilon}}{1-\varepsilon}+r_1 s(\tau)^{-\varepsilon}-\frac{\rho s(\tau)^{-\varepsilon}}{1-\varepsilon}+\frac{\pi'(\tau)}{\pi(\tau)}\frac{s(\tau)^{-\varepsilon}-p^{-\varepsilon}}{1-\varepsilon}\right) = \left(\rho\pi(\tau)-\pi'(\tau)\right)\omega
$$

and using the expression for a_τ we obtain an implicit expression for τ, as a function of a_0.

$$
\begin{aligned}
&e^{(1-\varepsilon)r_1\tau}\left[1+\frac{s(\tau)}{p}\left[e^{p\tau}-1\right]\right]^{\varepsilon-1} \\
&\qquad\times\frac{\varepsilon}{\varepsilon-1}s(\tau)^{-\varepsilon}\left(p-s(\tau)-\frac{1}{\epsilon}\frac{\pi'(\tau)}{\pi(\tau)}\left(1-\frac{p^{-\varepsilon}}{s(\tau)^{-\varepsilon}}\right)\right)=\pi(\tau)\left(\rho-\frac{\pi'(\tau)}{\pi(\tau)}\right)\omega a_0^{\varepsilon-1}
\end{aligned}
$$

The value function of the full program is:

$$
\begin{aligned}
W(a_0) &= \int_0^\tau \frac{\left(s(\tau)X(\tau)a_0 e^{\frac{r_1-\rho}{\varepsilon}t}\right)^{1-\varepsilon}}{1-\varepsilon}e^{-\rho t}dt+\varphi(\tau,a_\tau) \\
&= \frac{\left(s(\tau)X(\tau)a_0\right)^{1-\varepsilon}}{1-\varepsilon}\int_0^\tau e^{-pt}dt+\varphi(\tau,a_\tau) \\
&= \frac{\left(s(\tau)X(\tau)a_0\right)^{1-\varepsilon}}{1-\varepsilon}\left[\frac{-1}{p}e^{-pt}\right]_0^\tau+\varphi(\tau,a_\tau) \\
&= \frac{\left(s(\tau)X(\tau)a_0\right)^{1-\varepsilon}}{1-\varepsilon}\left[\frac{-1}{p}e^{-p\tau}+\frac{1}{p}\right]+\varphi(\tau,a_\tau)
\end{aligned}
$$

C Proofs

C.1 Proof of Proposition 2

For $r_2 \geq r_1$ and $\sigma = 0$, it is possible to show that $\frac{\partial W(a_0)}{\partial \tau} < 0$, thus proving the first part of the proposition.

To prove the second part of the proposition, we rewrite Equation (23) as (see Appendix B):

$$e^{(1-\varepsilon)r_1\tau}\left[1+\frac{s(\tau)}{p}\left[e^{p\tau}-1\right]\right]^{\varepsilon-1} \\ \times \frac{\varepsilon}{\varepsilon-1}s(\tau)^{-\varepsilon}\left(p-s(\tau)-\frac{1}{\epsilon}\frac{\pi'(\tau)}{\pi(\tau)}\left(1-\frac{p^{-\varepsilon}}{s(\tau)^{-\varepsilon}}\right)\right)=\pi(\tau)\left(\rho-\frac{\pi'(\tau)}{\pi(\tau)}\right)\omega a_0^{\varepsilon-1} \quad (31)$$

In the neighborhood of $\tau = 0$, since $\pi'(\tau) = 0$, it can be simplified to:

$$e^{(1-\varepsilon)r_1\tau}\left[1+\frac{s(\tau)}{p}\left[e^{p\tau}-1\right]\right]^{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1}s(\tau)^{-\varepsilon}\left(p-s(\tau)\right)=\pi(\tau)\rho\omega a_0^{\varepsilon-1}$$

The right-hand side of this equation is always positive and finite. For $r_2 = r_1$ and $\sigma = 0$, the left-hand side (LHS) of the equation is equal to zero. In addition, $\partial LHS(\tau)/\partial\sigma > 0$ and $\lim_{\sigma\to\sigma_{\max}} LHS \to \infty$ with $\sigma_{\max} = \left[2(r_2 - \rho/(1-\varepsilon)/\varepsilon\right]^{1/2}$. Therefore, there exists $\underline{\sigma} > 0$ such that $\sigma > \underline{\sigma} \Leftrightarrow \tau^* > 0$. This proves the second part of the proposition.

To prove the third part of the proposition, we compare the functions $W_1(a_\tau)$ and $W_2(a_\tau)$. We have $\lim_{\sigma\to\sigma_{\max}} W_2(a_\tau) \to -\infty$ for a finite a_τ, while $W_1(a_\tau)$ remains finite as it is not affected by σ. Therefore, $\lim_{\sigma\to\sigma_{\max}} \tau^* \to +\infty$. In addition, $\frac{\partial W_2(a_\tau)}{\partial\sigma} < 0$ and we know from the first part of the proposition that for $r_2 = r_1$ and $\sigma = 0$, it is optimal to get pregnant as soon as possible. Therefore, there exists a value $\bar{\sigma}$ of σ that is sufficiently large to have $\tau^* \geq T$ if $\sigma > \bar{\sigma}$.

C.2 Proof of Proposition 3

For $\tau \geq T$ and $\tau \leq 0$, we have $\pi'(\tau) = 0$. Under the assumption $\pi'(\tau) = 0$ and τ finite, the derivative of the LHS and of the RHS of Equation (31) with respect to τ and ω can be written:

$$\frac{\partial LHS}{\partial\tau} = e^{(1-\varepsilon)r_1\tau}\varepsilon s(\tau)^{-\epsilon}(p-s(\tau))\left(-r_1\left[1+\frac{s(\tau)}{p}\left(e^{p\tau}-1\right)\right]+s(\tau)e^{p\tau}\right)$$

$$\frac{\partial LHS}{\partial\omega} = 0, \frac{\partial RHS}{\partial\tau} = 0, \frac{\partial RHS}{\partial\omega} = \pi(\tau)\rho a_0^{\varepsilon-1} > 0$$

For $\rho < r_1$ and $\pi'(\tau) = 0$, the LHS is then a decreasing function for all τ which ensures $\bar{\omega} < \tilde{\omega}$. $\rho < r_1$ and τ in the neighborhood of 0 and T (which ensures $\pi'(\tau) = 0$) are sufficient, together with continuity, to yield the existence and uniqueness of $\bar{\omega}$ and $\tilde{\omega}$.

D Kernel Density Estimations of Income Growth Distribution - Alternative Samples

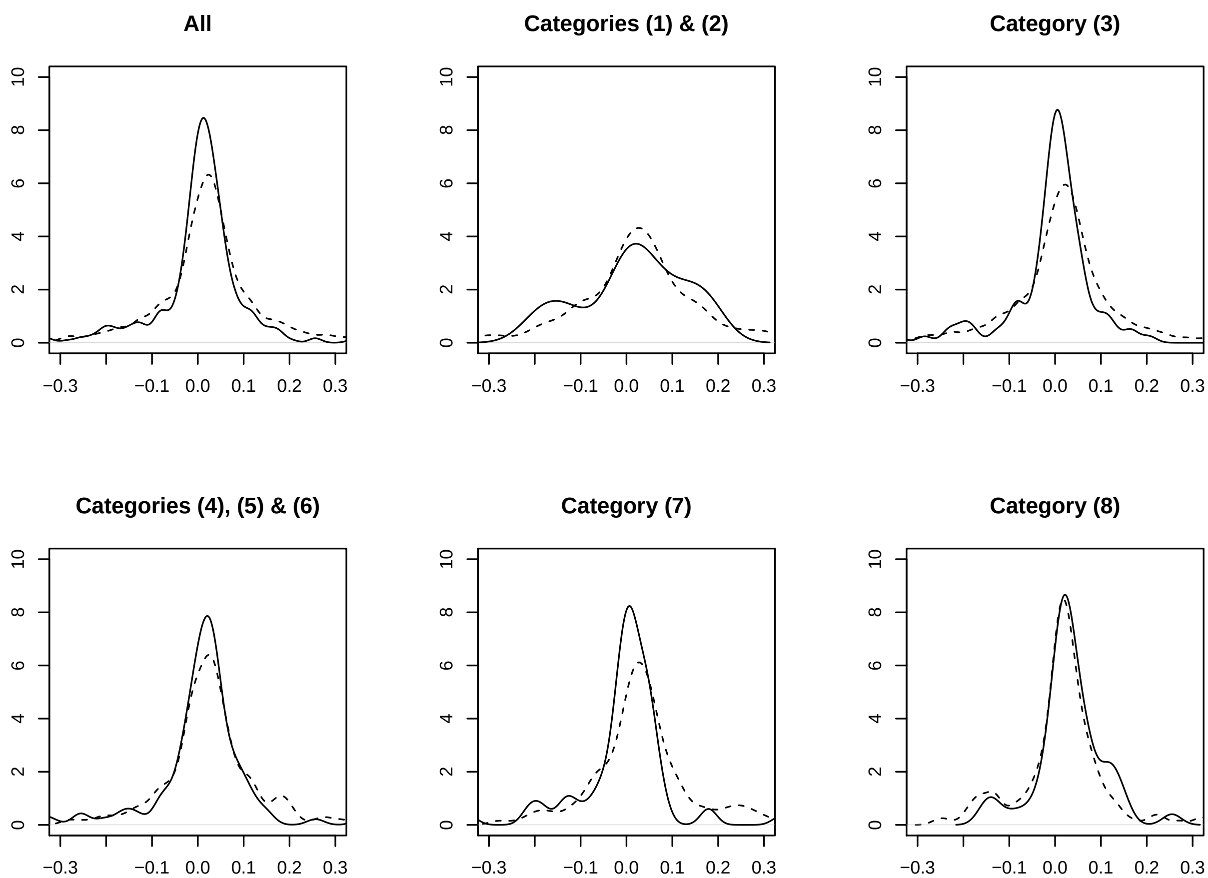

Figure 3: Kernel density estimations of individual income growth distribution by education category. Married women. Childless women (solid) and mothers (dashed)

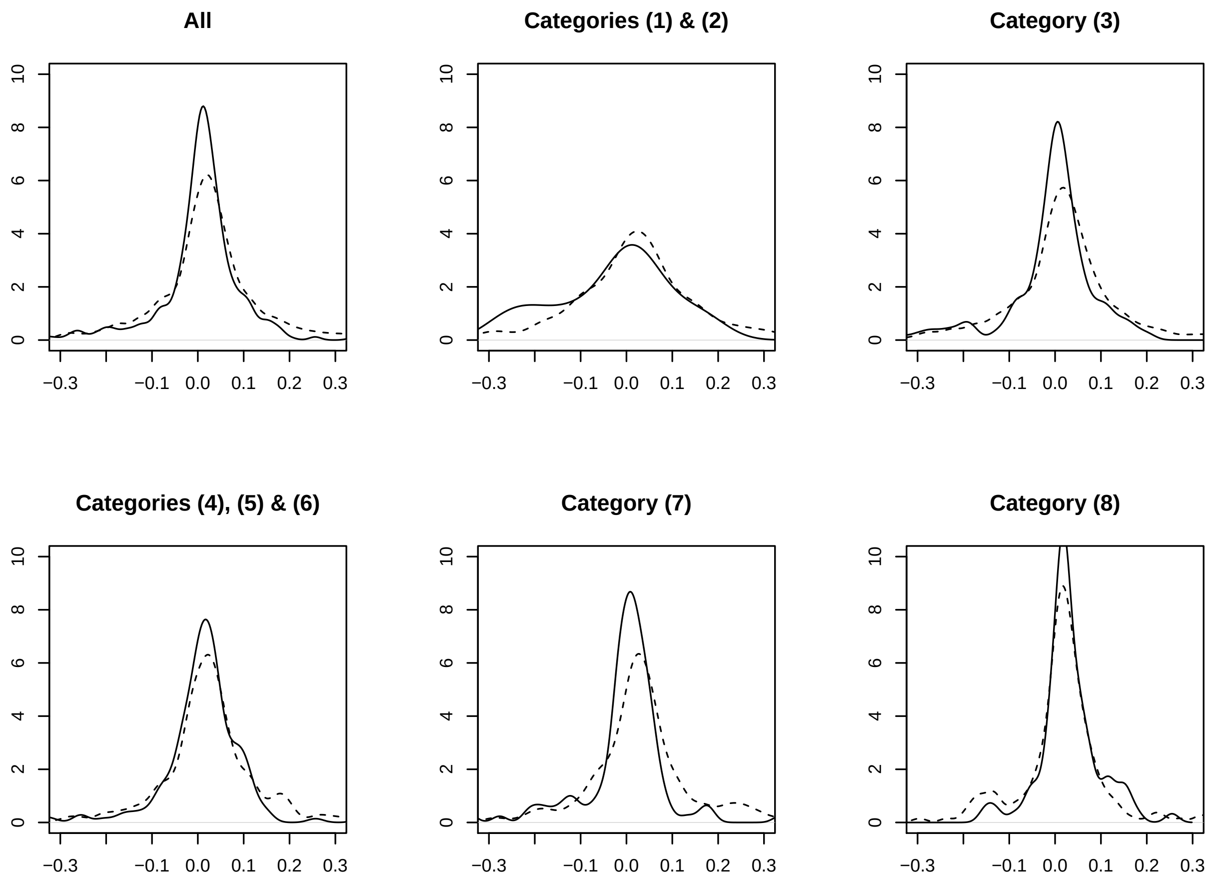

Figure 4: Kernel density estimations of individual income growth distribution by education category. All women except teenage (< 16) mothers. Childless women (solid) and mothers (dashed)